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FACIAL GROWTH IN CHILDREN OF FIVE TO EIGHT YEARS OF AGE

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INTRODUCTION

IRREGULARITIES of facial growth in the region of the jaws are common and much time and effort has been expended by members of the dental profession in classifying shades of variation found in the relationship of the mandible to the maxilla. Appliances have been devised to correct different types of mal-occlusion but orthodontists are often at a loss to know whether improvement achieved by such methods might not have appeared unassisted in the normal course of growth. Knowledge of the direction and rate of normal growth of the jaws is therefore required to permit dental surgeons to decide whether orthodontic treatment is necessary or not.

The late Dr. Matthew Young, in collaboration with Miss K. C. Smyth and others, made facial measurements on children at different ages, to establish standards of growth. Their first investigation, *Facial Growth in Children* (Smyth and Young, 1932), was directed, for the main part, to children aged 8 through 13 years who were *selected* for measurement in so far as only those were included in whom the two dental arches exhibited what is usually described and generally regarded

as the "normal" relationship to one another, the main indication of this condition being that the ridge of the mesio-buccal cusp of the upper first permanent molar occludes in the buccal groove of the lower first permanent molar. The selection was stringent and although several thousand candidates were reviewed only about 6 per cent were finally accepted. (In this connection Young's conclusion was that "while *morphologically normal* and *ideal* occlusion seem to be legitimate terms, it is certainly a misnomer to apply the term *normal,* without further definition, to a type of 'bite' which is found in less than 10 per cent of the general population of children, as was elicited in the present investigation.")

Similar measurements were also taken for a group of children aged 2 through 4 years. In this series *no selection* was made beyond the rejection of such as were unfit for measurement through extensive caries or other incidental circumstances, so that all types of occlusion were included.

The second investigation carried out by Young, Johnson, Smyth and Still (1937) was on the common form of mal-occlusion known as *"inferior post-normal occlusion,"* in which condition the *lower jaw appears to be situated further back than usual in relation to the upper;* this mal-occlusion is usually diagnosed from an inspection of the relative positions of the upper and lower first permanent molars. Facial measurements were made of children from 8 through 13 years of age and the averages were compared with those obtained in the previous investigation of children with "normal" or "ideal" occlusion. The main conclusion was that:

> "Contrary to the general belief, it would appear from the measurements here recorded that the disharmony observed in the jaws and dental arches is due more frequently to excessive forward growth of the upper jaw than to retrusion or undergrowth of the lower jaw."

THE PRESENT INVESTIGATION

These earlier observations related, it will be observed, to children of age 2 - 5 and 8 - 14 years and left a gap in the knowledge obtained between 5 and 8 years,—ages which correspond to the second "springing-up" period referred to by various authors and described by Harris in 1931. It has been recognised for many years that in childhood there are periods of accelerated growth which occur at approximately 2, 5

to 8, and 12 to 14 years of age; the third period is usually a year or so later in boys than in girls but the first and second appear at about the same ages in the two sexes. The 5 to 8 year age period should prove, therefore, to be an interesting and possibly valuable one during which to study facial growth, particularly as it is at times of rapid growth that defects are most easily produced and recognised.

The previous investigations had suggested that future work might be done more profitably by means of radiography and in the preface to the second report (1937) it was said that:

> "To obtain new knowledge regarding the actual site of excessive or defective growth, more accurate and more detailed measurements at successive intervals in individual children would be essential. The ideal method of extending the work would be to obtain standardised radiographs of the profile and face views of adequate numbers of children with normal and post-normal occlusion beginning at an early age and repeated at yearly or half-yearly intervals over several consecutive years. By this means much information, at present almost completely lacking, would be gained about the actual possibilities of normal individual growth and adjustment."

The present investigation was planned by Young on these lines as a radiographic study of the growth of an unselected group of children at ages 5 to 8 years, thus filling in the gap in existing data.

Arrangements were made by the late Professor H. H. Woollard with Dr. R. P. Garrow, Medical Officer of Health of Hornsey, to have 200 elementary school children of 5 through 7 years *selected at random* visit University College every six months. The advent of the war cut short the investigation and only 4 examinations were made, covering a period of 18 months. Even during this short period the original number of 200 children had dwindled considerably; at the end of the first year about 134 were still available and they are the children considered in this report as it is convenient to assess the growth of each age group for a one-year period.

These measurements had not been completed or analysed when both Dr. Young and Professor Woollard died. They seemed worthy to the present author, who had assisted Dr. Young, to be put on record and analysed in accordance with Dr. Young's intentions.

The children were first weighed and measured (without shoes) and their dental condition as to caries and structure (M-hypoplasia) was charted (Allen, 1941). Dr. Garrow kindly supplied particulars of home environment, illnesses suffered up to the time of the first inspection

and whether or not extra milk was taken at school. Lateral and full-face radiographs were then taken, under the supervision of Dr. Matthew Young, of the children's heads, wrists, and, at the first inspection only, of their feet. To secure facial measurements comparable from one examination to the next, and in all the children, the heads had to be orientated in as exact a position as possible in relation to the film; this was done by placing them in an instrument devised for the purpose, based on the apparatus originally designed by Broadbent (1931). Unfortunately the ear-plugs of the instrument were made of metal which threw an opaque shadow on the films and in many of them obscured the auditory meatus (see next section).

Method of measurement

Measurements were made on the radiographs (see Fig. 1) similar to those taken by Smyth, Still and Johnson on living children; the main ones considered here are those from the transmeatal axis (TA) to the following:

Nasion	(N)
Sub-nasal point	(SNP)
Upper incisor gum margin	(UIGM)
Upper incisor incisal margin	(UIIM)
Lower incisor incisal margin	(LIIM)
Lower incisor gum margin	(LIGM)
Mental point	(MP)
Sub-mental point	(SMP)

It was also decided by Dr. Young that two radial measurements should be taken from the centre of the pituitary fossa, one to the nasion and the other to the sub-nasal point.

Two vertical measurements were made from the nasion to the sub-nasal point and the upper incisor gum margin. It was thought that any other vertical measurements from the nasion would be valueless owing to the constant changes in the dentition caused by the shedding of deciduous and eruption of permanent teeth at these ages. Measurements were also made of the breadths of the face and dental arches and of the length of the mandible on the lateral film as well as of the gonial angle.

A difficulty arose in making the radial measurements from the transmeatal axis. In taking the radiographs the children's heads were held in position by metal ear-plugs which, as stated above, threw opaque shadows on the films and in many, though not all, of the lateral films obliterated the auditory meatus. In making such measurements it is

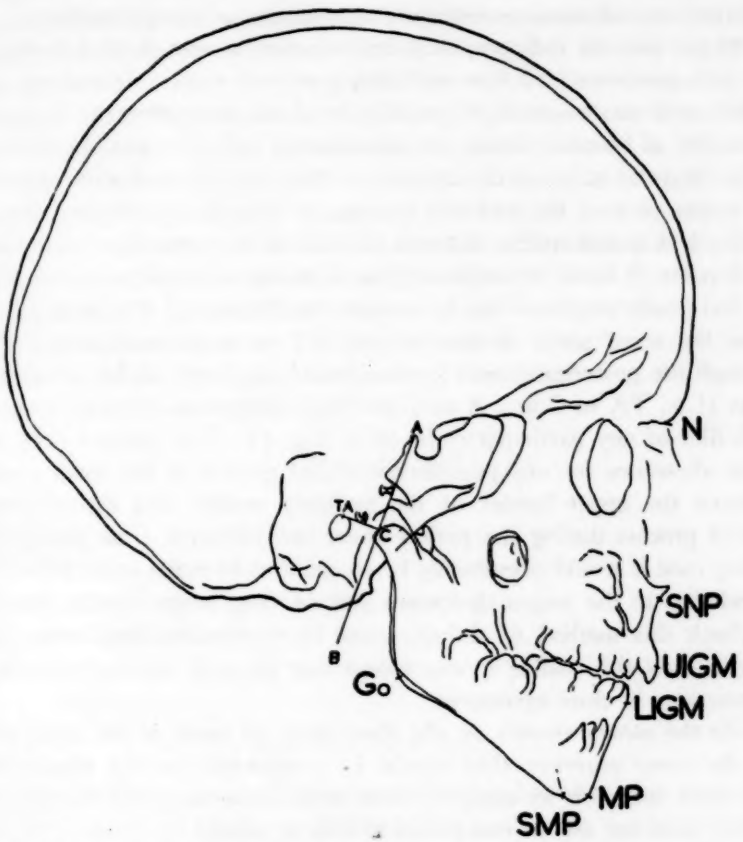

FIG. 1. TRACING OF A LATERAL RADIOGRAPH SHOWING THE POINTS OF MEASUREMENT

important that the points used should be *comparable* in different children as well as in the same child at successive inspections. A secondary consideration is that they should be comparable with methods used by other workers. Anthropological measurements can be taken from many

points and in many planes. Radiographic measurement is in the nature of pioneer work and sometimes different methods must be devised, as points which are well defined in a dry or a living skull may *not* be clearly defined in a radiograph and it is important to choose those which can be easily recognised and kept reasonably constant. If the methods are then clearly described it will be apparent to what extent different sets of measurements are comparable or complementary.

In the present radiographical investigation it was desired to bring the measurements into line with the previous work referred to and therefore it was essential, if possible, to obtain measurements from, or as nearly as possible from, the transmeatal axis. It was found that in the films of most of the children at least one of each child showed the upper part of the auditory meatus, so that if a point on it could be fixed at a measurable distance in relation to some other easily defined point, it could be arbitrarily fixed at the other inspections. The method finally employed was to measure the distance of this point (TA) from the fixed point of intersection (C) on a perpendicular (AB) through the posterior clinoid process, which is clearly visible on all the films (i. e., TA to C and A to C are fixed distances, (y) and (x), on each film of any particular child, as in Fig. 1). This method does not make allowance for any possible individual growth in the small region between the upper border of the auditory meatus and the posterior clinoid process during the period under consideration (one year), but in any case it would presumably be so small as to make little difference in relation to the longer diameters and, in fact, where it was possible to check this method of fixing points in consecutive films when TA was in each case visible it was found that the real and the calculated points were in close agreement.

As the measurements on the films were all made in the same way by the same observer they would be comparable among themselves, but when we wish to compare them with those made by Smyth and others there are one or two points to bear in mind:

(1) Smyth's measurements were made on living children and therefore from the cartilaginous (*not* the osseous) meatus to points on the face separated from the bone by soft tissues;

(2) The radiographic measurements have to be adjusted for magnification.

Concerning point (1) some useful information was available. In 1931 Young and Campion found that in 36 male subjects the osseous

axis lay, on the average, 2.63 mm. higher than the cartilaginous axis from the Frankfort plane; the osseous meatus was 0.84 mm. nearer the nasion and 2.13 mm. further from the mental point, although these last two differences were not significant owing to the relatively wide range of variability in the distances between the cartilaginous and osseous axes. It is seen, therefore, that facial measurements taken from radiographs can never be *absolutely* the same as those taken from apparently similar points on the living subject but the differences in the distances between the transmeatal axes probably balance the differences due to the presence and absence of soft tissues, and one might reasonably expect that the two sets of measurements would be sufficiently comparable to show any *trend* of growth, and, in fact, they do show closely comparable results. Moreover, the trend of growth shown by these measurements can be verified by considering those taken from the pituitary fossa to the nasion and the sub-nasal point as well as the measurements of the mandible.

From the measurements made on the lateral films the actual facial measurements were estimated by the use of a modification of the formula given by R. E. Roberts and W. H. Hooton:

$$\frac{a\left\{c-(\frac{1}{2}b+6.5)\right\}}{c}$$

where

a = measurement on film,
b = thickness of head (between ear-plugs),
c = focus-film distance (5 ft. = 152.4 cm.),
6.5 = distance (in cm.) of film from nearer side of head.

ANALYSIS OF RESULTS

The children were not examined clinically for occlusion nor were the films taken with this condition in mind, the aim being to obtain serial radiographs showing growth in an unselected group of children. It was only when measurements were begun on the films that differences in occlusion were particularly noticed and it was thought advisable in making some preliminary calculations to divide the children into groups in relation to normal and mal-occlusion and the results are given in Tables 1 to 10 and compared diagrammatically in Figs. 2 to 5. This sub-division reduces the numbers in the groups but gives interesting

results which agree with the figures obtained previously for younger and older children. It must be emphasised that the degree of post-normality is often slight and certainly less than that taken by Smyth in the previous work,—sometimes the lower jaw was only far enough back to bring the 1st permanent molars (or 2nd deciduous molars) into a cusp-to-cusp relationship; moreover the children had not always fully closed their teeth when the films were taken, so that the diagnosis from the films is by no means perfect; nevertheless it is based on three (and sometimes four) consecutive films and it is improbable that the same abnormal (i. e., non-habitual) relative position of the jaws would be maintained by a child four times at intervals of six months.

It is, of course, realised that the numbers available for this investigation are too small for detailed statistical analysis but taken in conjunction with the previous investigations it is thought that they give indications of the *trend of growth*. There were 19 boys and 17 girls who were between 5 and 6 years of age at the first inspection and who were seen three times. Of these 19 boys only 8 appeared to show the type of occlusion commonly recognised by dental surgeons as "normal" or "ideal"; 5 showed some degree of mandibular post-normality, 3 showed pre-normal occlusion (a condition in which the mandible appears to be in a forward position in relation to the maxilla) and in 3 cases the molars were in a post-normal relation while the incisors appeared to be slightly pre-normal. Of the 17 girls, 13 showed "normal," 3 post-normal and 1 pre-normal occlusion.

At age 6 there were 34 boys of whom 14 showed "normal," 15 post-normal and 2 pre-normal occlusion and the remaining 3 had the post-normal-molar-pre-normal-incisor relationship. Of the 25 girls at this age, 12 had "normal," 11 post-normal, 1 pre-normal and 1 post-normal-molar-pre-normal-incisor occlusion.

At age 7 there were 23 boys and 16 girls; of the boys 11 showed "normal" and 11 post-normal occlusion and 1 had the post-normal-molar-pre-normal-incisor relationship. Of the 16 girls, 7 had "normal," 8 post-normal and 1 pre-normal occlusion.

It may be well to explain here that the children were grouped according to age at their last birthday; thus in the tables, "5-" means 5 years up to, but not including, 6 years. This group in the text is referred to simply as "5 years." Such a group as "8 - 14 years" has been described as "8 through 13 years," a concise form of expression familiar to United States' readers.

From Table 1 it will be seen that in each age group for the boys and at ages 6 and 7 for the girls less than half showed the so-called "normal" type of dental occlusion even when the classification was less stringent than that adopted by Smyth.

TABLE 1

Percentage distribution of types of occlusion

AGE	TOTAL IN GROUP	NORMAL	POST-N	PRE-N	POST-PRE
		Boys			
5-	19	42%	26%	16%	16%
6-	34	41	44	6	9
7-	23	48	48	—	4
		Girls			
5-	17	77	18	6	—
6-	25	48	44	4	4
7-	16	44	50	6	—

BOYS

Antero-posterior measurements of the face

Averages were first calculated for all the boys with the type of occlusion usually considered "normal" or "ideal," as already described. (The term "normal" will be used in this sense throughout this paper for the sake of convenience.) These averages were compared with those obtained by Smyth for unselected children of 2-5 years and children of 8-14 years showing normal occlusion (Table 2); the measurements taken from radiographs at ages 5 through 7 fit in very well between those for ages 4 and 8 taken on the living subject; the only marked exception is TA to N and the reason for this difference is that it is impossible to identify the nasion (i. e., the middle of the naso-frontal suture) accurately on the living face. There is also a slight amount of overlapping in the measurements to the gum margin, particularly the lower where the measurement at age 6 from the radiographs is the same as at age 8 on the living subject and at age 7 is

TABLE 2

Comparison of antero-posterior measurements (in mm.) of the face made on living children at ages 2 -5 and 8 - 14 years (Smyth) and from radiographs at 5 - 8 years in children with normal occlusion

AGE	TA to N	TA to UIGM	TA to UIIM	TA to LIGM	TA to MP
			Boys		
2 -	78.5	75.0	76.1	73.0	82.2
3 -	80.3	78.5	79.4	76.8	86.8
4 -	81.6	79.2	80.4	78.7	88.9
5 -	**76.8**	**80.3**	**81.0**	**81.2**	**88.7**
6 -	**78.3**	**84.3**	**85.2**	**85.8**	**95.2**
7 -	**81.1**	**85.9**	**86.8**	**88.2**	**96.4**
8 -	86.2	85.6	87.1	85.8	97.6
9 -	88.0	88.3	90.5	88.7	101.2
10 -	89.0	88.7	90.9	89.4	101.9
11 -	89.3	89.4	91.6	90.1	103.0
12 -	90.7	91.3	93.0	91.5	105.0
13 - 14	92.2	92.8	94.3	93.6	108.8
			Girls		
2 -	77.0	75.8	76.5	74.6	84.4
3 -	79.1	76.8	77.8	75.8	85.2
4 -	79.7	78.9	80.0	79.1	90.2
5 -	**75.7**	**79.9**	**80.2**	**81.1**	**89.3**
6 -	**76.8**	**81.3**	**82.2**	**84.2**	**92.3**
7 -	**79.7**	**84.9**	**85.9**	**87.4**	**94.9**
8 -	84.9	84.1	85.7	84.5	96.3
9 -	84.6	85.0	87.2	85.7	98.0
10 -	86.5	86.3	88.4	86.9	99.6
11 -	87.8	88.2	89.8	88.5	101.5
12 -	89.0	89.4	91.4	89.7	103.5
13 - 14	90.1	90.8	92.8	92.0	107.4

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Fig. 2.

actually greater. This is probably due in part to the erupting state of the incisors at the earlier ages and also to the fact that the gum margin is not clearly defined on the films and a point has to be taken on the teeth which is judged to correspond to the gum margin. It is interesting to note the considerably greater amount of growth between ages 5 and 8 in the lower measurements than in the upper and also the greater amount of growth at this time than at any other age period recorded.

The average measurements in boys who showed normal occlusion can be compared with those of the boys showing post-normal occlusion, though it must be noted that the observations are few in number. At age 5 the average radial face measurements are all greater in the post-normals than in the normals (see Table 3 and Fig. 2). At age 6 the

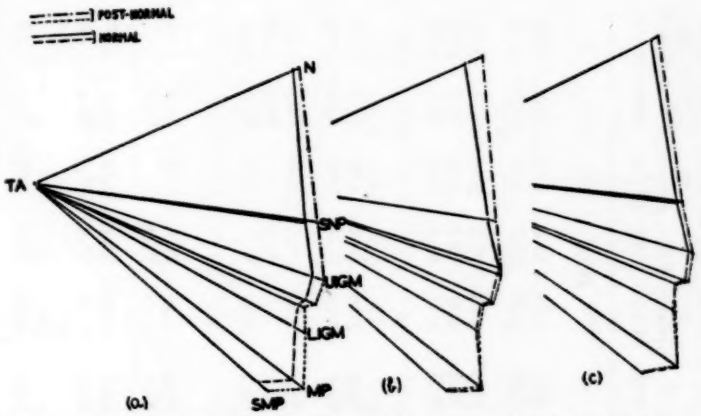

FIG. 2. DIAGRAMMATIC REPRESENTATION (ONE-HALF ACTUAL SIZE) OF THE ANTERO-POSTERIOR AND VERTICAL MEASUREMENTS OF THE FACE IN BOYS WITH NORMAL AND POST-NORMAL OCCLUSION AT (a) 5 YEARS, (b) 6 YEARS AND (c) 7 YEARS

N. B. Of the *vertical* distances only N to SNP and UIGM are exact measurements.

average measurements to the upper face are still larger in the post-normals but those to the mandible are the same size as in the normals. These figures together with the annual percentage rates of growth between 5 and 6 years suggest that at this age the growth from TA to the mandible falls behind in these post-normals, or rather fails to increase its rate of growth as might be expected judging by the marked increase in this region in the normals at this age. At age 7 the measure-

TABLE 3

Antero-posterior measurements of the face

GROUP	NO. IN GROUP	TA to N mm.	TA to N % growth in 1 yr.	TA to SNP mm.	TA to SNP % growth in 1 yr.	TA to UIGM mm.	TA to UIGM % growth in 1 yr.	TA to UIIM mm.	TA to UIIM % growth in 1 yr.	TA to LIIM mm.	TA to LIIM % growth in 1 yr.	TA to LIGM mm.	TA to LIGM % growth in 1 yr.	TA to MP mm.	TA to MP % growth in 1 yr.	TA to SMP mm.	TA to SMP % growth in 1 yr.
							Boys										
Age 5-																	
Normal	8	76.8	3.09	74.5	2.99	80.3	2.85	81.0	2.85	79.5	2.83	81.2	3.57	88.7	4.23	82.0	4.40
Post-normal	5	78.2	4.75	78.0	3.63	83.2	3.98	83.7	4.20	81.2	2.73	84.6	2.23	92.5	3.13	85.6*	—
Post-n molars, pre-n inc.	3	77.6	2.33	74.5	2.13	75.9	3.75	76.8	3.15	80.0	1.65	81.2	1.65	87.7	1.55	81.9	1.10
Pre-normal	3	78.2	3.93	76.6	2.00	81.2	2.27	82.2	3.33	80.7	3.80	82.8	4.10	89.0	4.17	81.9	5.67
All cases	19	77.6	3.49	75.8	2.81	80.8	3.15	81.5	3.35	80.1	2.85	82.2	3.09	89.4	3.57	82.2	3.80
Age 6-																	
Normal	14	78.3	2.31	78.1	2.31	84.3	3.19	85.2	4.09	83.1	2.29	85.8	2.47	95.2	2.96	88.6	3.22
Post-normal	15	82.5	1.72	80.3	1.70	85.5	2.10	86.1	3.03	83.9	2.05	86.0	2.43	94.8	2.41	88.2	2.68
Post-n molars, pre-n inc.	3	78.5	1.57	76.6	2.80	81.2	2.63	81.6	2.60	81.9	5.05	83.7	4.40	92.6	4.00	85.8	4.70
Pre-normal	2	74.5	2.50	74.1	3.10	79.6*	—	80.0*	—	80.5	1.75	82.8	1.15	89.7	1.55	83.3	1.15
All cases	34	79.9	2.01	78.7	2.16	84.3	2.65	84.9	3.39	83.2	2.34	85.6	2.50	94.4	2.74	87.9	3.00
Age 7-8																	
Normal (a)	11	81.1	1.64	80.2	2.03	85.9	2.29	86.8	2.79	85.2	2.06	88.2	1.74	96.4	2.74	90.1	3.00
" (b)	25	80.5		80.0		86.0		87.3		85.0		88.0		96.2		89.8	
Post-normal (a)	11	81.5	2.56	80.7	1.87	87.3	2.03	88.1	2.71	85.7	2.28	87.2	2.15	96.3	2.06	89.6	1.65
(b)	26	82.8		81.3		87.3		88.4		85.4		87.5		96.2		89.8	
Post-n molars, pre-n inc.	1																
All cases (a)	23	81.2	2.21	80.5	1.90	86.5	2.16	87.3	2.78	85.5	2.23	87.9	1.97	96.2	2.38	89.8	2.27

* One case only.

TABLE 3 *(Continued)*

Antero-posterior measurements of the face

* One case only.

TABLE 3 *(Continued)*

Antero-posterior measurements of the face

GROUP	NO. IN GROUP	TA to N mm.	TA to N % growth in 1 yr.	TA to SNP mm.	TA to SNP % growth in 1 yr.	TA to UIGM mm.	TA to UIGM % growth in 1 yr.	TA to UIIM mm.	TA to UIIM % growth in 1 yr.	TA to LIIM mm.	TA to LIIM % growth in 1 yr.	TA to LIGM mm.	TA to LIGM % growth in 1 yr.	TA to MP mm.	TA to MP % growth in 1 yr.	TA to SMP mm.	TA to SMP % growth in 1 yr.
							Girls										
Age 5-																	
Normal	13	75.7	1.94	74.7	1.86	79.9	2.22	80.2	2.65	79.4	2.47	81.1	2.91	89.3	3.27	83.4	3.17
Post-normal	3	75.2	5.10	74.5	4.45	80.3	4.65	81.2	5.25	79.1	2.35	80.7	4.60	90.2	2.60	83.7	3.30
Pre-normal	1																
All cases	17	75.6	2.47	74.5	2.22	79.8	2.70	80.2	3.22	79.4	2.45	81.1	3.22	89.4	3.12	83.4	3.20
Age 6-																	
Normal	12	76.8	1.51	76.6	2.11	81.3	2.56	82.2	3.36	81.6	1.73	84.2	2.93	92.3	2.12	85.6	2.07
Post-normal	11	78.8	1.95	77.8	1.95	82.9	3.11	83.7	3.67	81.9	2.80	84.4	3.13	92.5	2.76	86.0	2.83
Post-n molars, pre-n inc.	1																
Pre-normal	1																
All cases	25	78.0	1.77	77.3	2.01	82.1	2.73	83.0	3.34	82.2	2.21	84.6	2.87	92.7	2.34	86.1	2.29
Age 7-8																	
Normal (a)	7	79.7	1.49	79.0	1.34	84.9	1.70	85.9	2.46	85.2	1.70	87.4	1.60	94.9	1.30	89.0	1.40
" (b)	19	78.6		78.6		84.2		85.7		84.1		86.8		94.8		88.3	
Post-normal																	
(a)	8	80.5	1.16	79.7	1.14	85.3	1.80	86.7	2.13	83.5	1.08	86.2	1.28	93.1	2.50	87.0	2.40
(b)	19	80.3		79.6		85.5		87.0		83.8		86.6		94.6		88.1	
Pre-normal	1																
All cases	16	79.9	1.39	79.2	1.56	85.0	1.94	86.2	2.53	84.5	1.75	86.8	0.19	94.1	1.88	88.1	1.92

ments to the upper face are again larger in the post-normals though the differences at the nasion and sub-nasal point are small. The measurements from TA to the mandible do not show any marked difference between the two groups but when they are compared with the two previous age groups and with the yearly percentage rate of growth between 6 and 7 years they do suggest that in the post-normals growth in the mandibular region is relatively slower than in the normals. As the numbers are small further averages were calculated for the 7 year age group by adding to them the measurements for the 6 year group taken at the end of one year (i. e., 3rd inspection). It will be seen that these figures (series b) confirm the greater size of the upper face in the post-normals and show that the measurements from TA to the mandible are the same size for both groups at this age.

In the earlier clinical investigation, from ages 8 through 13 in boys the post-normals maintained and increased the greater size of the upper face and although there is a suggestion that the mandible (TA to LIGM and TA to MP) was smaller the differences are not always clear cut, possibly due to the fact that different observers were concerned in taking these measurements. The evidence so far considered suggests that in boys the difference in growth between the "normals" and "post-normals" consists of greater early growth in the post-normals followed by relatively slower growth from 5 years onwards when the greatest impetus in normal growth is found in the lower half of the face (TA to LIGM and TA to MP). Turning from the comparison between normals and post-normals we may study the growth rate of each separately throughout the 5 - 8 year period. The measurements have been represented diagrammatically in Fig. 3. The normals present a picture of smooth, regular growth; between 5 and 6 the increase in size is progressively greater downwards from the nasion to the mental point. In the post-normals the growth pattern is very uneven and there is little increase in size in the mandible region.

Comparison of measurements from TA and PF to N and SNP

Comparing the measurements from the centre of the pituitary fossa (PF) to the nasion and sub-nasal point (Table 4), there is in the normals a picture of even, outward and downward growth and the increases between the age groups are approximately proportionate to the increases shown in the measurements to the nasion and sub-nasal point

taken from TA. In the post-normals, however, the yearly increases are small and irregular and do not at all correspond to the increases in the measurements from TA to the nasion and sub-nasal point. If these measurements are compared in Table 5 it is seen that in the normals between age 5 and 7 (series b) there is an increase of 3.7 mm. in TA to N and 2.3 mm. in PF to N (the latter being 62.2 per cent of the former). In the post-normals the corresponding increases are 4.6 mm. and 1.5 mm. (the latter being 32.6 per cent of the former). Similarly, comparing the measurements to the sub-nasal point (SNP) from TA and PF, the increases in the normals are 5.5 mm. and 4.1

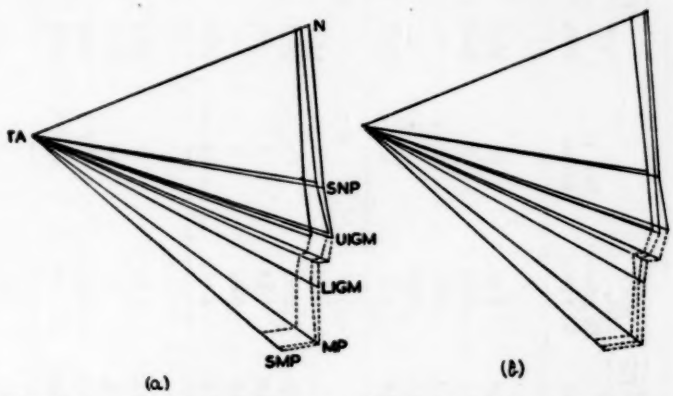

FIG. 3. DIAGRAMMATIC REPRESENTATION (ONE-HALF ACTUAL SIZE) OF THE ANTERO-POSTERIOR AND VERTICAL MEASUREMENTS OF THE FACE IN BOYS AT AGES 5, 6 AND 7, WITH (a) NORMAL AND (b) POST-NORMAL OCCLUSION

N. B. Of the *vertical* distances only N to SNP and UIGM are exact measurements.

mm. (the latter being 74.5 per cent of the former) and in the post-normals 3.3 mm. compared with 1.2 mm. (the latter being 36.4 per cent of the former.) Further, we can compare the percentage rates of growth by taking the increase between the 5 year group and the 7 year group and expressing it as a percentage of the first. In the normals the percentage rates of growth from TA to N and to SNP are approximately the same as the corresponding ones from PF but in the post-normals the percentage increases from PF are less than half those from TA.

The measurements from TA and PF are approximately, though not exactly, in the same plane but the differences are sufficiently large

TABLE 4

Measurements from the centre of the pituitary fossa (PF) to the nasion and sub-nasal point

GROUP	BOYS					GIRLS				
		PF to N		PF to SNP			PF to N		PF to SNP	
	No. in group	mm.	% growth in 1 year	mm.	% growth in 1 year	No. in group	mm.	% growth in 1 year	mm.	% growth in 1 year
AGE 5-										
Normal	8	57.1	2.29	64.8	2.43	13	56.0	1.47	64.9	2.72
Post-normal	5	58.3	3.93	67.2	2.73	3	54.6	5.05	64.7	4.25
Post-n molars, pre-n inc.	3	55.8	1.67	61.4	2.97	—	—	—	—	—
Pre-normal	3	56.8	3.80	65.6	2.30	1	—	—	—	—
All cases	19	57.2	2.83	65.0	2.57	17	55.8	1.99	64.9	2.97
AGE 6-										
Normal	14	57.6	3.07	67.5	2.16	12	55.7	1.64	65.8	2.48
Post-normal	15	59.5	2.12	67.6	2.09	11	56.9	2.52	66.6	1.91
Post-n molars, pre-n inc.	3	57.0	1.57	66.0	1.83	1	—	—	—	—
Pre-normal	2	55.2	2.50	64.9	2.80	1	—	—	—	—
All cases	34	58.2	2.51	67.3	2.14	25	56.4	1.96	66.0	2.14
AGE 7-8										
Normal (a)	11	59.4	1.57	68.9	1.80	7	58.3	2.01	67.7	2.56
" (b)	25	59.4	—	68.9	—	19	57.2	—	67.5	—
Post-normal (a)	11	58.4	3.16	67.5	2.35	8	58.4	1.36	67.1	1.59
" " (b)	26	59.8	—	68.4	—	19	58.4	—	67.5	—
Post-n molars, pre-n inc.	1	—	—	—	—	—	—	—	—	—
Pre-normal	—	—	—	—	—	1	—	—	—	—
All cases (a)	23	59.3	2.46	68.3	2.07	16	58.1	1.57	67.2	2.22

All cases (a)	23	59.3	2.46	68.3	2.07	16	58.1	1.57	67.2	2.22

TABLE 5

Growth in measurements TA and PF to N and SNP

	NORMALS				POST-NORMALS			
	TA to N	TA to SNP	PF to N	PF to SNP	TA to N	TA to SNP	PF to N	PF to SNP
				Boys				
Age 5-6								
Average mm.	76.8	74.5	57.1	64.8	78.2	78.0	58.3	67.2
Age 7-8 (b)								
Average mm.	80.5	80.0	59.4	68.9	82.8	81.3	59.8	68.4
Increase-mm.	3.7	5.5	2.3	4.1	4.6	3.3	1.5	1.2
" %	4.8	7.4	4.0	6.3	5.9	4.2	2.6	1.8
				Girls				
Age 6-								
Average mm.	76.8	76.6	55.7	65.8	78.8	77.8	56.9	66.6
Age 7-8 (b)								
Average mm.	78.5	78.6	57.2	67.5	80.3	79.6	58.4	67.5
Increase-mm.	1.7	2.0	1.5	1.7	1.5	1.8	1.5	0.9
" %	2.2	2.6	2.7	2.6	1.9	2.3	2.6	1.4

to suggest that in the post-normals a greater proportion of the growth between 5 and 8 years occurs in the region shown in radiographs between and around TA and PF (see Fig. 1). The comparison of the percentage rates of growth appears to confirm this conclusion.

Vertical measurements of the face

Turning to the vertical measurements from the nasion, it is seen that at ages 5 and 6 (Table 6) N to SNP is greater in the post-normals; at age 7 it is slightly smaller in the post-normals in series (a) but in the bigger series (b) it is still slightly larger although even here the difference in size between normals and post-normals is considerably reduced compared with ages 5 and 6. The percentage rates of growth at the earlier ages also would suggest that the normals are growing slightly faster in this region than the post-normals.

The measurement N to UIGM at 5 and 6 is greater in the post-normals while at age 7 in series (a) there is no difference between the two averages; in series (b) the post-normals were still larger than the normals although the difference between the two groups is less than at the earlier ages. This measurement was not made at the later ages in the earlier clinical investigations.

As has already been noted, the nasion as taken on the living subject is not comparable with the more accurate identification from radiographs, so that the actual size of the two sets of measurements cannot be compared but the relative differences between normals and post-normals would suggest that at all ages the measurement N to SNP is greater in the post-normals. The measurements from the nasion to UIIM, $\overline{6|6}$ (i.e., the upper first permanent molars) and SMP at ages 8 - 14 are all smaller in the post-normals; at these ages the depth of the mandible (LI to SMP and LM to LBM) does not differ significantly although it is, if anything, slightly greater in the post-normals. It would appear, therefore, that the shortening of the face, if real, in the post-normal boys occurs between the sub-nasal point and the molar occlusal plane and first begins between about 6 and 8 years of age. It may be noted that during the 5 - 8 years period the permanent incisors are erupting; when measuring the radiographs there seemed to be a suggestion of shortening between the sub-nasal point and the upper incisor gum margin but owing to this eruption the incisor gum margin was very difficult to determine on the films and the results correspondingly un-

TABLE 6

TABLE 6

Vertical measurements of the face

GROUP	BOYS					GIRLS				
	No. in group	N to SNP mm.	N to SNP % growth in 1 year	N to UIGM mm.	N to UIGM % growth in 1 year	No. in group	N to SNP mm.	N to SNP % growth in 1 year	N to UIGM mm.	N to UIGM % growth in 1 year
AGE 5-										
Normal	8	41.3	5.44	56.7	4.87	13	41.6	6.11	56.9	4.23
Post-normal	5	42.5	4.88	58.3	4.23	3	43.9	1.95	58.0	1.50
Post-n molars, pre-n inc.	3	40.2	3.83	52.4	9.05	—	—	—	—	—
Pre-normal	3	43.2	4.27	58.2	2.60	1	—	—	—	—
All cases	19	41.8	4.82	56.9	4.80	17	42.0	5.35	56.9	3.70
AGE 6-										
Normal	14	42.9	4.68	56.9	4.05	12	42.8	3.98	56.5	2.60
Post-normal	15	45.4	3.32	60.4	2.31	11	44.8	2.60	57.5	2.94
Post-n molars, pre-n inc.	3	43.2	4.97	58.6	—	1	—	—	—	—
Pre-normal	2	40.0	3.55	54.3	—	1	—	—	—	—
All cases	34	43.9	4.08	58.5	3.16	25	43.9	3.30	57.3	2.74
AGE 7-8										
Normal (a)	11	44.8	4.09	58.7	2.38	7	43.1	4.04	56.1	2.13
" (b)	25	44.9	—	58.8	—	19	44.0	—	56.8	—
Post-normal (a)	11	43.8	4.40	58.7	3.51	8	44.3	3.53	56.8	3.00
" (b)	26	45.4	—	60.0	—	19	45.3	—	58.0	—
Post-n molars, pre-n inc.	1	—	—	—	—	—	—	—	—	—
Pre-normal	—	—	—	—	—	1	—	—	—	—
All cases	23	44.3	4.25	58.5	3.23	16	43.8	3.67	56.2	2.71

satisfactory. It was decided not to attempt to measure N to UIIM on the radiographs as the measurement would be of no value until the incisors were fully erupted. For the same reason, no measurements were made on the radiographs of the depth of the mandible.

The mandible

It was possible to supplement the measurements already considered by others of the mandible itself:

Condyle to gonion	(Con. to Go.)
" to MP	(Con. to MP)
Gonion to MP	(Go. to MP)
Gonial angle	

In taking radiographs of the skull it is very difficult, if not impossible, to show different parts of it with equal clarity on the same film owing to their varying densities and in many of the films in this investigation the heads of the condyles are rather obscure and in some cases quite undefinable. The numbers measured are therefore very small and little stress can be laid on the figures obtained from them. It will be seen in Table 7, however, that the averages obtained confirm the differences already shown between "normals" and "post-normals." At age 5 the post-normals are bigger in every respect; the smallest difference (0.8 mm.) being in the measurement (Con. to Go.) while the difference in the total length of the mandible (Con. to MP) is considerable (4.4 mm.). At age 6 the depth of the ramus (Con. to Go.) is still greater in the post-normals but the length of the mandible (Con. to Go.) and (Go. to MP) is now greater in the normals, owing to the spurt of growth which occurred in the mandible in the normals between 5 and 6 years. At age 7 the main difference in both series (a) and (b) would appear to be in the length from Go. to MP, which is longer in the normals. The total mandibular length is slightly shorter in the post-normals (Con. to MP) as shown by the longer series (b) but not by series (a).

The only measurements of the mandible made in the previous investigations on older children were from the gonion to MP and LIGM. No consistent differences were found between normals and post-normals and the former measurement does not appear to be comparable with the similar one taken from the radiographs which is considerably smaller;

TABLE 7
Measurements of the mandible

GROUP	NO. IN GROUP	CON. TO GO. mm.	CON. TO GO. % growth in 1 year	CON. TO MP mm.	CON. TO MP % growth in 1 year	GO. TO MP mm.	GO. TO MP % growth in 1 year	GONIAL ANGLE deg.	GONIAL ANGLE change in 1 year
				Boys					
AGE 5-									
Normal	8	40.1	5.75	80.7	3.65	52.4	4.52	132.1°	+1.17°
Post-normal	5	40.9	4.53	85.1	3.90	54.7	5.63	135.5°	
Post-n molars, pre-n inc.	3	38.2		81.9		51.1		137.0°	
Pre-normal	3	44.4		84.4		51.5		132.3°	
All cases	19	40.8	4.59	82.6	3.49	52.7	4.98	133.6°	+0.47°
AGE 6-									
Normal	14	42.0	4.06	87.3	1.89	56.8	3.89	132.8°	+0.50°
Post-normal	15	43.3	4.24	85.1	3.19	55.2	3.82	131.6°	—0.73°
Post-n molars, pre-n inc.	3	41.4		81.6		54.3		129.3°	
Pre-normal	2	38.2		81.0		51.5		134.0°	
All cases	34	42.3	4.58	85.2	3.03	55.5	3.80	132.0°	—0.10°
AGE 7-8									
Normal (a)	11	44.6	6.76	87.7	4.13	57.9	2.74	128.6°	+1.29°
" (b)	25	44.1		88.2		58.1		131.3°	
Post-normal (a)	11	43.8	5.45	87.6	3.83	55.6	3.81	131.2°	+0.50°
" " (b)	26	44.2		87.4		56.4		131.3°	
Post-n molars, pre-n inc.	1								
All cases (a)	23	43.9	5.81	87.7	3.92	56.8	3.34	130.3°	+0.78°
				Girls					
AGE 6-									
Normal	12	40.8	2.94	84.4	3.21	53.8	3.61	134.8°	+0.09°
Post-normal	11	41.9	2.69	85.1	2.24	55.7	3.44	130.6°	—0.09°
All cases	25	41.2	2.80	85.0	2.78	54.9	3.50	133.0°	+0.43°
AGE 7-8									
Normal (a)	7	41.7	3.07	85.8	2.32	57.7	2.88	129.0°	+0.50°
" (b)	19	41.5		86.3		56.5		131.4°	
Post-normal (a)	8	41.7	3.54	85.1	1.38	53.2	4.90	133.7°	—1.60°
" " (b)	19	42.7		86.0		55.9		131.7°	
All cases (a)	16	41.8	3.37	85.7	1.86	55.4	3.76	131.2°	—0.42°

the difference must presumably be accounted for by the soft tissues involved in measuring from the gonion on the living face and also possibly by the different methods of identifying the gonion on the curved angle of the mandible.

If the growth of the mandible is examined in normals and post-normals separately between 5 and 8 years it is seen that in the normals there is a considerable spurt of growth between ages 5 and 6 from both the condyle and the gonion to the mental point (Con. to MP and Go. to MP) while the gonial angle remains the same. Between ages 6 and 7 there is slight general growth, greatest in the condyle to gonion region and the gonial angle is slightly reduced. In the post-normals the greatest growth occurs between the condyle and gonion (Con. to Go.) and that mainly between ages 5 and 6. There is very little growth between the gonion and mental point in the three years and an increase of only 2.5 mm. in Con. to MP compared with 7 mm. in the normals. There is some reduction in the gonial angle between ages 5 and 6.

The data suggest that in the first instance boys of 5 years showing some degree of mandibular post-normality had generally greater antero-posterior facial measurements than others showing the so-called "normal" occlusion; subsequently they failed to exhibit the greatly accelerated mandibular growth normally occurring around 6 years so that, while maintaining the relatively greater size of the upper face, the mandible became the same size as (if not actually smaller than) that in the "normal" group. There is furthermore a suggestion that in the "post-normal" boys a relatively large proportion of the growth between ages 5 and 8 years occurred between and near the points TA and PF.

GIRLS

Averages were first calculated for the girls showing "normal" occlusion and these were compared with those obtained by Smyth for an unselected sample of girls of ages 2 - 5 and selected girls of 8 - 14 years showing "normal" occlusion (Table 2). As with the boys the measurements from the radiographs agree well with those on the living subject with the same exceptions, namely TA to N and slight overlapping of ages 7 and 8 years in the gum region.

Whereas in the boys there was a very much greater spurt of growth between ages 5 and 6 than in any other year and the increase in the size of the antero-posterior measurements of the face was at the same

time progressively greater downward from nasion to mental point, in the girls, although the growth between 5 and 8 years is rather greater than at any other period, the difference between the increases in TA to MP and in TA to N is smaller than in boys; moreover, the accelerated growth is spread more evenly over the 2-year period in girls than in boys, where it is mainly concentrated between 5 and 6 years.

Antero-posterior measurements of the face

Among the girls at age 5 there were 13 showing normal occlusion but only 3 showing post-normal occlusion, so little can be said about this age group; but, as can be seen from Table 3 and Fig. 4, all the

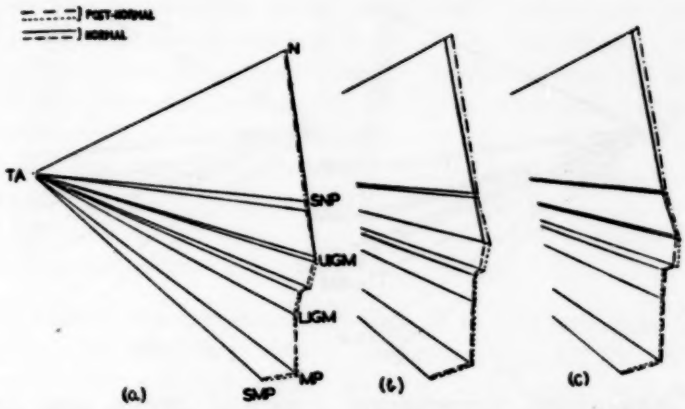

FIG. 4. DIAGRAMMATIC REPRESENTATION (ONE-HALF ACTUAL SIZE) OF THE ANTERO-POSTERIOR AND VERTICAL MEASUREMENTS OF THE FACE IN GIRLS WITH NORMAL AND POST-NORMAL OCCLUSION AT (a) 5 YEARS, (b) 6 YEARS AND (c) 7 YEARS

N. B. Of the *vertical* distances only N to SNP and UIGM are exact measurements.

antero-posterior dimensions of the face are about the same in both groups.

Judging by the annual percentage rates of growth, the post-normals in the age 5 group (only 3 in number) appear to be growing faster than the normals. It should be noted, however, that in the post-normal girls as in the boys the growth from TA to the mandible is relatively less than that of the upper face whereas the reverse is true of the normals.

At age 6 the antero-posterior measurements of the upper face are appreciably larger in the post-normals but those to the mandible (LIIM, LIGM, MP and SMP) are the same in both groups. Between ages 6 and 7 the annual percentage rates of growth appear to be about the same in normals and post-normals.

At age 7 all the measurements from TA to N, SNP, UIGM and UIIM are slightly larger in the post-normals while those from TA to the mandible are smaller in the post-normals in series (a) but the same in series (b).

In the girls at ages 8 - 14 years (measured by Smyth and Johnson) there was no difference between the average measurement TA to N

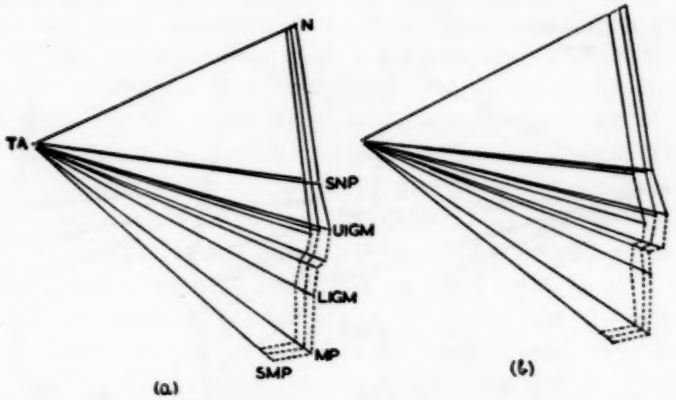

FIG. 5. DIAGRAMMATIC REPRESENTATION (ONE-HALF ACTUAL SIZE) OF THE ANTERO-POSTERIOR AND VERTICAL MEASUREMENTS OF THE FACE IN GIRLS AT AGES 5, 6 AND 7, WITH (a) NORMAL AND (b) POST-NORMAL OCCLUSION

N. B. Of the *vertical* distances only N to SNP and UIGM are exact measurements.

in normals and post-normals. However, TA to UIGM and TA to UIIM were distinctly bigger throughout in the post-normals whereas the measurements TA to LIGM and TA to MP were smaller in the post-normals; all these differences were large enough to be statistically significant (with one or two exceptions in certain age groups).

In Fig. 5 is shown diagrammatically the increases in the antero-posterior measurements in the three age groups in the normals and in the post-normals. The former gives a picture of regular growth through the whole face, and contrasted with the boys there is not the same greatly increased growth to the mental point. The growth picture

for the three post-normal groups of girls is not so irregular as that for the boys, though it again appears to be relatively deficient in the mandibular region.

Comparison of measurements from TA and PF to N and SNP

Table 4 shows the increases in the measurements from PF to the nasion and sub-nasal point. In the normals there is very little growth between 5 and 6 but more between 6 and 7 years. In the post-normals there is quite a considerable amount of growth at each period and this is also at variance with the findings for the boys.

In Table 5 figures are given to compare the increases in the measurements from PF and TA to N and SNP between ages 6 and 7. (The youngest group is omitted owing to the small numbers available.) It will be seen that the increases to N from TA and PF are about the same in both normals and post-normals. In the normals TA to SNP and PF to SNP show increases of much the same order, whereas in the post-normals PF to SNP shows a rate of growth which is only about half that shown by TA to SNP. This agrees with the similar results for boys.

Vertical measurements of the face

The vertical measurements of the face, N to SNP and N to UIGM, are both larger at all three ages in the post-normals but the difference between normals and post-normals for N to SNP appears to be slightly decreasing owing to the faster growth in a vertical direction of the normals.

It is not possible to compare the vertical heights of the face in the girls from 8 - 14 years as the two observers do not appear to have determined the nasion in a comparable manner.

The mandible

The numbers of girls, as of boys, available for measurements of the mandible are very small and little stress can be laid on the results. Age 5 is omitted altogether (see Table 7).

At age 6 all the measurements of the mandible, Con. to Go., Con. to MP and Go. to MP, are greater in the post-normals and the gonial angle is slightly smaller. The percentage rates of growth between 6

and 7 are slightly greater in the normals and at age 7 the measurements Con. to MP and Go. to MP are slightly greater for the normals; Con. to Go. appears to be about the same (series a) or slightly smaller (series b) in the normals and the gonial angles are probably about the same.

The site of the greatest growth in the normals appears to be Go. to MP; the increase in Con. to MP is rather less.

BREADTHS OF THE FACE AND JAWS

Boys

As the measurements from the full face films have not been adjusted for magnification it is not possible to compare the actual size on the films with that found from clinical measurements. The relation of normals to post-normals can however be compared as the amount of magnification would not have varied appreciably from one film to another. (See Table 8.)

At ages 5, 6, and 7 on the radiographs the post-normals have consistently greater bizygomatic and bigonial breadths, the differences in the latter measurements being fully 3 mm. At the later ages, 8 - 14 years, in the clinical investigations the normals appeared to have the greater breadths, but as it was shown that the two observers taking the measurements varied in these by between 2 and 3 mm. the comparison is perhaps not very reliable.

Comparing the breadths of the dental arches, there seems to be very little difference between post-normals and normals in the widths between upper deciduous canines and between lower deciduous canines; the most marked differences are in the upper arch, where the normals are 0.9 mm. *greater* at age 5 and 0.5 mm. *less* at age 7. At age 6 there appears to be no difference in the breadths of the dental arches between the upper or lower first permanent molars; at age 7 the post-normals seem to have slightly greater width in the lower arch but there is no difference between the groups in the upper arch. At ages 8 - 14 the previous observers found that the widths of the dental arches between the first permanent molars were greater in the normals and these differences were more marked in the upper arch than in the lower (at age 8, 2.7 mm. in uppers compared with 1.68 mm. in lowers).

TABLE 8
*Breadths of the face and jaws**

GROUP	NO. IN GROUP	B at c|c	B at $\overline{c\|c}$	B at 6|6	B at $\overline{6\|6}$	BIZYGOMATIC BREADTH	BIGONIAL BREADTH
				Boys			
AGE 5-							
Normal	8	34.9	29.3			113.4	81.3
Post-normal	5	34.0	29.7			114.4	84.6
Post-n molars, pre-n inc.	3	33.5	28.5			110.3	79.3
Pre-normal	3	36.0	29.7			114.3	85.0
All cases	19	34.6	29.3			113.3	82.4
AGE 6-							
Normal	14	35.7	29.5	55.6	57.2	116.5	83.0
Post-normal	15	35.4	29.8	55.3	57.4	119.5	86.7
Post-n molars, pre-n inc.	3	34.2	28.5	54.5	54.8	118.3	81.7
Pre-normal	2	32.0	26.5	—	—	110.5	76.5
All cases	34	35.2	29.4	55.3	57.0	117.7	84.2
AGE 7-8							
Normal (a)	11	36.1	30.9	55.4	56.5	118.0	84.6
" (b)	25	36.3	30.5	55.7	56.5	118.3	84.7
Post-normal (a)	11	36.9	30.6	56.7	58.1	119.6	87.6
" " (b)	26	36.8	30.5	55.9	57.8	120.7	87.8
All cases (a)	23	36.7	30.8	56.1	57.3	118.5	86.1
				Girls			
AGE 6-							
Normal	12	34.9	29.7	54.2	55.3	114.8	83.3
Post-normal	11	35.4	30.1	53.4	54.9	116.3	82.8
All cases	25	35.0	29.9	53.6	55.2	115.6	83.3
AGE 7-8							
Normal (a)	7	36.5	31.0	55.0	56.6	115.4	83.0
" (b)	19	36.0	30.8	55.4	55.6	116.7	84.0
Post-normal (a)	8	35.6	30.6	54.1	55.3	118.4	85.1
" " (b)	19	36.4	30.9	54.3	55.5	118.3	85.0
All cases (a)	16	35.8	30.8	54.4	55.7	117.1	84.3

* These measurements have not been adjusted for magnification on the films.

Girls

At ages 6 and 7 in the girls the bizygomatic breadth was greater in the post-normals than in the normals (Table 8). The bigonial breadth was approximately the same size in both groups at 6 and very slightly greater in the post-normals at 7. None of the differences was as great as those found in the boys. At 8 - 14 years in the earlier investigation these breadths appeared to be always greater in the normals, but as mentioned for the boys there seemed to be greater variability in the taking of these breadths by the different observers than in any of the other measurements and the comparison is therefore less satisfactory.

There are no marked differences in the widths between the deciduous canines, upper or lower; at age 6 the post-normals are slightly wider and at age 7 they are slightly narrower. In the widths between first permanent molars the normals seem to be slightly greater than the post-normals in both upper and lower arches at age 6 and 7, the differences being slightly more marked at the later age. At ages 8 - 14 in the earlier investigation it was found that the normals had dental arch widths between the first permanent molars which were considerably wider than in the post-normals and the differences were greater in the upper arch than in the lower (at 8 years, 4.03 mm. upper compared with 2.3 mm. lower).

BOYS COMPARED WITH GIRLS

At age 5 the measurements from TA are of much the same size for boys as for girls with normal occlusion (except TA to N which is always larger in boys), whereas the measurements for post-normal boys are generally larger than for the corresponding girls. The vertical measurements from the nasion are much the same for both sexes. At ages 6 and 7 all the measurements for boys (both post-normal and normal) are greater than those for the corresponding girls except the vertical measurements (N to SNP and UIGM) in the normals at 6, where they remain about the same size.

Comparing the measurements of the mandible, at age 6 all measurements for normal boys are greater than for normal girls; in post-normals there are no differences except for Con. to Go. which is greater in boys. At age 7 all measurements for boys are greater than

for corresponding girls but the differences are smaller in the post-normals.

Comparing the size of the radiographic measurements (unadjusted for magnification) of the breadths of the face and jaws in boys and girls at ages 6 and 7 it is seen that:

(1) All bizygomatic breadths are wider in boys than in the corresponding group of girls.

(2) At 6 years the bigonial breadths are the same in normal boys and girls but wider in post-normal boys than in post-normal girls. At 7 years, these breadths appear to be slightly wider in normal boys than girls but the difference is more marked in the post-normal boys.

(3) In the breadths between the upper canines there is no great difference between boys and girls in either normals or post-normals.

(4) In the breadths between the lower canines there are no differences between boys and girls.

(5) The breadths between first permanent molars in the boys seem rather larger than in the girls at 6 in both normal and post-normal groups; at age 7 the breadths are all wider in boys than in girls with the exception of the upper breadth in the normals which is the same in boys as in girls.

OTHER DATA

Boys

It may be suggested that some of the most common irregularities in occlusion of the maxilla and mandible should be treated as a problem related to a disturbance of the rhythm of general growth of the body and not as one concerning the jaws alone. It has frequently been remarked that children showing post-normal occlusion appear to be above the average in physique, and in the clinical investigations carried out by Smyth and Young it was found that on the average they were considerably taller at all ages than children with normal occlusion. In the present investigation (Table 9) it was found that at ages 5 and 6 the boys with post-normal occlusion had greater average heights than those with normal occlusion (differences of 5.3 cm. and 3.3 cm. at the two ages), but at age 7 there was no difference between normals and post-normals.

The post-normals had a greater area of carpal bones developed at all ages, the difference being greatest at age 5.

TABLE 9
Miscellaneous data

GROUP	NO. IN GROUP	HEIGHT Average (cm.)	HEIGHT % growth in 1 year	WEIGHT / HEIGHT	CARPAL BONES Average total area (mm.²)	CARPAL BONES % growth in 1 year	PITUITARY FOSSA Av. area (mm.²)	AV. NO. TEETH PRESENT Decid.	AV. NO. TEETH PRESENT Perm.	AVERAGE CARIES FIG. AT FIRST INSPECTION	AV. NO. INFECTIVE FEVERS UP TO FIRST INSPECTION
					Boys						
AGE 5-											
Normal	8	107.8	5.9	17.7	208	36.9	44.5	18.5	—	0.72	1.25
Post-normal	5	113.1	5.6	18.0	311	37.1	40.8	20.0	—	0.37	1.20
AGE 6-											
Normal	14	116.7	5.3	19.0	297	39.6	42.0	14.5	4.3	1.26	2.07
Post-normal	15	120.0	5.0	19.6	331	36.1	46.6	15.7	3.4	1.13	2.40
AGE 7-8											
Normal	11	122.2	4.6	18.7	406	35.3	44.0	13.9	5.8	0.87	2.44
Post-normal	11	122.0	4.9	19.3	444	30.0	48.3	12.4	6.9	1.02	2.09
					Girls						
AGE 6-											
Normal	12	116.6	5.6	18.9	386	32.3	40.8	14.8	5.2	1.09	2.08
Post-normal	11	117.0	5.6	18.6	425	28.6	42.6	11.6	6.5	1.31	2.27
AGE 7-8											
Normal	7	119.7	4.7	17.8	421	26.9	42.0	12.7	6.7	1.13	2.43
Post-normal	8	120.8	4.6	18.4	496	26.8	42.1	10.0	6.5	1.31	1.89

In view of these results and accepting the fact that normal growth of the face proceeds outwards and downwards and that the upper face (TA to N) approaches its maximum size at an earlier age than the mandible, it would appear that the boys with post-normal occlusion have exhibited greater general growth than the normals at an early age. There is, of course, the possibility that this greater early growth was excessive (i. e., abnormal) and in this connection it is interesting to examine the data available for other conditions.

In addition to their greater height and carpal bone development, at all three ages the weight/height index was very slightly higher in the post-normals. At age 5 the post-normals had more deciduous teeth present in the mouth (20.0 compared with 18.5) and a lower average caries figure (0.37 : 0.72). At age 6 the post-normals still had a higher average number of deciduous teeth present in the mouth (15.7 : 14.5) but there is no difference in the average caries figures (1.13 : 1.26). At age 7 the post-normals had fewer deciduous teeth present (12.4 : 13.9) but more permanent (6.9 : 5.8).

It will be noticed that for many of these conditions the differences between normals and post-normals become a little smaller as age increases.

Girls

Omitting comparison of the age 5 groups as only 3 girls showed post-normal occlusion, it will be seen from Table 9 that at ages 6 and 7 there is little difference in the average heights and weight/height indices for normals and post-normals but the average total carpal bone area developed is greater in the post-normals; they have fewer deciduous teeth than the normals (11.6 : 14.8 at age 6, and 10.0 : 12.7 at 7) but more permanent (6.5 : 5.2) at age 6 and the same number at 7 years; the post-normals have slightly higher average caries figures at both ages.

Area of pituitary fossa on lateral head films

Information of interest was also available as to the size of the pituitary fossa in these children. The area was traced off the radiograph, measured with a planimeter and the measurement adjusted for magnification. In the boys the area of the fossa seemed to be smaller in post-normals at age 5 but greater at 6 and 7, whereas in girls there

was only a small difference at 6 years and none at all at 7 years. Moreover, in the boys there seemed to be a suggestion of increase in size with age, but not in the girls. To study this point further, averages at half-yearly intervals were calculated for all the children and all the inspections available (Table 10); to increase the numbers, the 3rd inspections of the 5-year-olds were added to the 6-year group, and the 4th inspections to the 6½-year group and so on for the 6 and 7 year age groups. Averages were first calculated for all boys and all girls irrespective of occlusion, and it was seen that the average size of the area increased steadily in boys from 42.2 sq. mm. to 48.7 sq. mm. while in girls it did not increase but remained at approximately 43.5 sq. mm. throughout the whole period from 5 years to 8½ years. Next, separating the groups into normal and post-normal, in boys there was a steady increase in the middle age groups (6-8 years), where the numbers are bigger, in both normals and post-normals, though the increase was greater in the normals which, nevertheless, had a smaller pituitary fossa area throughout the two-year period. In girls there was a consistent increase in the area in the post-normals but no consistent change in the normals.

DISCUSSION

In view of the high percentage of children who show some degree of post-normality of the mandible:

	Age 5-	Age 6-	Age 7-8
Boys	26 per cent	44 per cent	48 per cent
Girls	18 per cent	44 per cent	50 per cent

is it not a possibility that this relationship is to a certain extent part of a normal phase of growth?

In the clinical investigations it was found that the lengths of the upper and lower dental arches (i. e., the segment occupied by the ten deciduous teeth or their successors) do not increase between 2 and 14 years. This has been recognised by many other workers from John Hunter onwards (*Med. Res. Coun. Report,* No. 171, p. 40-43). The present radiological work suggests that the direction of facial growth from 5 to 8 years is outwards and downwards. These two facts taken in conjunction seem to suggest that owing to the greater relative growth of the mandible compared with the upper face from, say, 5 years on-

TABLE 10

The average area of the pituitary fossa (in sq. mm.) measured from lateral radiographs of the head

AGE GROUP	BOYS						GIRLS					
	Total group		Normals		Post-normals		Total group		Normals		Post-normals	
	No.	*mm.²*	*No.*	*mm.²*	*No.*	*mm.²*	*No.*	*mm.²*	*No.*	*mm.²*	*No.*	*mm.²*
5 -5½	16	42.15	8	43.82	5	40.85	13	43.38	12	44.32	3	39.86
5½-6	16	43.24	6	44.46	5	42.50	12	44.00	8	45.43	3	39.86
6 -6½	47	44.00	20	42.23	19	46.63	38	43.43	22	43.65	13	42.32
6½-7	45	45.82	18	43.90	19	47.11	36	43.63	21	43.64	12	43.08
7 -7½	51	46.65	23	45.40	24	48.69	41	43.40	19	42.90	20	43.42
7½-8	53	47.94	22	47.43	26	48.55	36	44.32	18	44.10	17	44.65
8 -8½	20	46.07	8	45.20	11	47.33	16	42.78	7	40.61	8	44.85
8½-9	19	48.7	10	48.21	8	48.65	16	43.07	7	40.48	8	45.20

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wards, the lower dental arch must in time be pushed forward in relation to the upper, which would convert a post-normal relationship into a "normal" one. But if that relatively increased growth of the mandible failed to appear the post-normality would remain for life. Similarly, if there was inadequate growth in early life when growth is relatively greater in the upper face (to N, SNP, etc.), the subsequent increased growth of the mandible might produce a pre-normal mandibular relationship. To study this further we should need to know whether a deficiency in the early growth of the face can be made up later.

Sillman (1942), in a study of mal-occlusion in the deciduous dentition, concluded: "Babies at birth have a posterior relationship of the jaws ranging from 0 to 8 mm. The mandible must grow forward, move forward, or both to the extent of 2 to 13 mm. more than the maxilla."

Brodie (1940) made a study of serial records of the growth of the face in 21 boys beginning at the age of 3 months and continuing for 8 years. He superimposed tracings of successive radiographs of individual children and studied trends of growth; his paper does not give any grouped mean measurements, which might have been compared with the present results. He concludes, "The chief finding is that very early growth is generalized; that is, all the surfaces seem to be increasing in size. At relatively early ages this picture dies out and there are left only certain sites which continue for varying periods thereafter." In 1941 he carried this conclusion further: "Adverse constitutional factors operating during the various stages of development can affect only the sites or processes active at that time . . . The effects of an arrest do not seem to be compensated for by a subsequent acceleration of growth to the affected part."

There seem to be some differences in the findings for boys and girls; whereas in boys the post-normals were originally bigger children who failed to maintain their higher standards (assuming the excessive growth is not an abnormality), in the girls the post-normals appear to have been of a slightly lower standard. Although the facial measurements for normals and post-normals of both sexes at ages 5 to 8 show a similar trend of growth on the whole, the differences in the upper face measurements between the two groups (i. e., normal and post-normal) are smaller in girls than in boys; this was also true for the

older children examined in earlier investigations and is perhaps related to the relatively smaller amount of growth in the girls.

SUMMARY AND CONCLUSIONS

(1) Measurements were made on lateral and antero-posterior radiographs of the heads of 134 boys and girls aged 5 through 7 years. The measurements corresponded as nearly as possible to those made in previous investigations by Young, Smyth, Still and Johnson.

(2) Some degree of post-normality of the mandible was found in 26 per cent of boys at age 5, 44 per cent at age 6, 48 per cent at age 7 years, and 18 per cent, 44 per cent, and 50 per cent in girls at similar ages. For this reason measurements for "post-normal" and "normal" children were kept separate, although the original intention was to study facial growth in an unselected group of children.

(3) The radiographic measurements for normal children at ages 5 through 7 years fit in well between those made by Smyth *et al.* on normal children at ages 4 and 8. A greater amount of growth occurred in the lower measurements of the face than in the upper (e. g., in TA to MP compared with TA to N); also the amount of growth between 5 and 8 years was greater than at any other age period recorded. Boys showed greater growth between 5 and 8 years than girls.

(4) In boys, at age 5 all measurements are greater in post-normals than in normals (there were only 3 girls showing post-normal occlusion at this age); at ages 6 and 7 the measurements to the upper face (TA to N, SNP and UIGM) in both boys and girls are all a little greater than in normals but those to the mandible are the same in both groups. The differences are more marked in boys than in girls; this fact is probably associated with their more rapid growth at these ages.

The radiographic data, taken in conjunction with previous clinical findings, suggest that the difference in growth between normals and post-normals consists of greater early growth in post-normals followed by relatively slower growth from 5 years onwards when the greatest impetus in normal growth is found in the lower half of the face (TA to LIGM and MP).

(5) Measurements were also taken from the centre of the pituitary fossa (PF) to N and SNP. In the normal boys these show fairly similar trends and rates of growth to those from TA. In the post-normals however the yearly increases are small and irregular; the in-

crease in size between 5 years and 7 years in PF to N and in PF to SNP is a little less than half that found in TA to N and in TA to SNP respectively. A similar result was found for girls for PF to SNP and TA to SNP but not for TA' and PF to N. This suggests that a greater proportion of the growth between ages 5 and 8 years in post-normal children, particularly in boys, occurs in the region between and around TA and PF.

(7) Vertical measurements were made from N to SNP and UIGM and at all three ages (5, 6 and 7), in boys and in girls both measurements were greater in post-normals than in normals, though the difference appears to be decreasing with age owing to the faster growth in a vertical direction of the normals.

(8) Measurements of the mandible were made from the condyle and gonion to the mental point and from the condyle to gonion. At age 5 the post-normal boys were bigger in every respect than the normals; at age 6 and 7 the length of the mandible had become greater in the normals. Between 5 and 8 years the increase in Con. to Go. was 7 mm. in normals compared with 2.5 mm. in post-normals and in Go. to MP, 5.5 mm. compared with 1.7 mm.

Similar trends were found in the girls at 6 and 7 years but the numbers are very small and little stress can be laid on the results; the differences appear to be smaller than in boys.

(9) Both bizygomatic and bigonial breadths in boys were slightly greater in post-normals than in normals; in girls the bigonial breadth appeared to be slightly greater in post-normals but there were no differences in bizygomatic breadth. There did not appear to be any differences in breadths of the dental arches in boys, but in girls the breadths between the first permanent molars seemed to be slightly greater in the normals.

(10) Post-normal boys were taller than the normals at 5 and 6 years but not at 7. The girls showed little or no differences in height.

(11) In both boys and girls the post-normals had a greater average total area of carpal bone developed than the normals.

(12) At age 7 in both boys and girls the post-normals had retained fewer deciduous teeth and had more permanent teeth erupted.

(13) Taking all boys and all girls together from 5 to 9 years irrespective of occlusion, it was found that in boys the area of the pituitary fossa (PF) as measured from lateral radiographs increases steadily in size from 42 to 49 sq. mm. while in girls it remains stationary at

about 43·5 sq. mm. Then dividing groups into those with normal and post-normal occlusion it was found that in both groups of boys there was a steady increase in the size of the area from 6 - 8 years though the increase was greater in the normals, who however had a smaller pituitary fossa area throughout this period. In girls there was a consistent increase in the area of the pituitary fossa in the post-normals from 5 to 9 years, but no consistent change in the normals.

(14) The possibility is discussed that some degree of post-normality of the mandible may be part of a normal phase of growth in view of (a) its prevalence, (b) the recognised fact that the segment of the dental arches occupied by the ten deciduous teeth and their successors does not increase in length between 2 and 14 years and (c) the relatively greater growth forward of the mandible at the later ages compared with the upper part of the face.

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A STUDY OF THE PALMAR DERMATOGLYPHICS OF MONGOLOID IMBECILES

BY DORIS MANN SNEDEKER

THE dermatoglyphics of mongoloid imbeciles present interesting and outstanding differences from those of normal individuals. Previous independent studies by Cummins ('39) and Workman ('39) on 139 mongoloids agree in all essential respects. In the palm and finger patterns of mongoloid imbeciles there are distinct trends which characterize the group. Cummins found that certain palmar patterns occur with greater frequency among mongoloid than among normal persons. The patterns found in mongoloid imbeciles are an exaggeration of the patterns found in the right hands of normal persons, and therefore might be considered as an exaggerated dextral trend. These disturbances in patterns must have occurred before the 12th to 14th week of fetal life, for it is during that period that the development of the dermal ridges is completed. Workman found a positive correlation of .21 to .36 between pattern expression and mongoloid imbecility.

The present study is concerned, first, in discovering how the dermatoglyphics of mongoloid imbeciles differ from those of normal persons; second, in determining the degree of significance of those differences; and third, in determining whether or not the dermatoglyphics of parents of mongoloids show any features which might have a diagnostic value. This study includes data from 173 mongoloid imbeciles.

Many suggested explanations of Mongolism have been made, such as syphilis, mental or physical suffering of the mother during pregnancy, advanced maternal age, and uterine exhaustion. Macklin made an extensive study of the literature and concluded that none of these suggestions are significant in the production of mongoloid imbecility. She suggested that it is inherited as a simple recessive character. Several investigators have reported multiple incidence of Mongolism in the same sibship, and in all cases the parents were normal. There are also records of two mongoloids each with a normal spouse having

a total of three normal children. This with other evidence strongly suggests recessive inheritance. However, Waardenburg and Penrose suggest that mongols may arise from chromosomal aberrations (Gates '46).

It appears that all the evidence except the usual occurrence of a single case in a family leads to the conclusion that Mongolism is a simple recessive, although the age of the mother may have some effect. The high frequency of single cases may be explained if we assume that the gene for Mongolism is of low penetrance and that in some cases maternal age may play a part in its manifestation.

The data for this study were obtained from the patients at the Institution for the Feebleminded at Columbus, Ohio and at the Orient State Schools at Orient, Ohio. In all, 173 individuals, 92 males and 81 females, were studied. As nearly as could be ascertained none were related. The ages of the patients ranged from 6 to 63 years. The age factor is not important in this study, however, since the patterns remain unchanged after their formation during embryological development.

The case histories of the subjects were studied in order to obtain the birth order in the sibship, the age of the mother at the time of the birth of the mongoloid child, the mental and chronological ages of the patient, and any other information which seemed pertinent.

The patients were observed for the following cardinal signs of Mongolism: epicanthus of the eye, fissured tongue, dark, straight hair, ear anomalies, and flat face and nose. It was also observed that a large number of the palms were small and flat. Crookshank describes the palms as "lax, plump, square and stubby; the digits being short, thick, somewhat tapering, and the minimus often incurved." The diagnosis of Mongolism had been made by the physician in charge. The patients used in the study were largely of European-American stock, although the prints of three Negroes were included.

In this analysis true as well as vestigial patterns were included. Vestigial patterns lack the sharp recurvatures of ridges which distinguish true patterns, but were included inasmuch as their presence indicates a genetic potentiality for development.

The data for the dermatoglyphics of normal European-Americans were taken from Cummins' study made in 1926.

The accompanying photograph (Fig. 1) will serve to illustrate the topography of the palm and the dermatoglyphic areas. The interdigital areas, I, II, III, IV, are numbered in sequence beginning with

the expanse between the thumb and index finger. The thenar eminence occupies the proximo-radial area of the palm, and the hypothenar eminence lies in the ulnar area of the palm. At the base of each digit are located the digital tri-radii, from which the palmar main lines are traced. The four main lines are named in radio-ulnar sequence, A, B, C, D. The formulation of main lines is based upon a numbered sequence of 14 positions around the periphery of the palm, each main line being described by the position in which it ends. The numbers for the termination of the main lines make up the main line formula. It is recorded in the order D, C, B, A, with periods separating the symbols. The method of classification described here is taken from Cummins' revised methods. A reliable test of the method would be desirable for there are some points which are open to personal variation in interpretation.

TABLE 1

Frequency of pattern occurrence in different palmar areas

PATTERN AREAS	MONGOLOID IMBECILES			NORMAL
	Cummins (97 palms)	Workman (153 palms)	Snedeker (346 palms)	European-Americans (400 palms)
Hypothenar	50.8%	75.8%	58.2%	37.2%
Thenar/I	3.0	5.2	1.6	5.5
II	14.8	13.1	5.4	4.2
III	80.6	76.4	95.5	40.8
IV	20.2	34.0	19.4	49.0

The three sets of data presented in Table 1 show agreement in the greater frequency of patterns in the hypothenar, second, and third interdigital areas of mongoloids than those occurring in normal persons, and lesser frequency of patterns in the thenar/first and fourth interdigital areas.

The variation in the three sets of data may be attributed to several causes:

(1) Both Cummins' and Workman's samples were small, each being under 100;

FIG. 1. DERMATOGLYPHIC AREAS AND SYMBOLS

(2) In several instances Cummins and Workman were not able to use both hands of the individual, whereas my study included only those individuals in which both hands were used; and

(3) There may be differences of opinion as to the classification of mongoloid imbeciles. In the three studies the diagnoses recorded in hospitals were accepted. Mongolism has many manifestations and although it is usually recognizable at birth and is no longer confused with cretinism, some patients seem less "typical" than others. Frequently some of the palm prints are also less typical.

In general, right hands of mongoloids show a distinct trend toward well-developed patterns, while small loops and vestiges occur more frequently in left hands.

There is a marked bi-manual asymmetry of patterns and vestiges in European-Americans in the fourth interdigital area, the left hands having the larger number. In the mongols this asymmetry is very

TABLE 2

Frequency of termination on main line D in various positions

POSITION	PARENTS AND SIBS (48 palms)	MONGOLS (346 palms)	EUROPEAN-AMERICANS (400 palms)
7	13.8%	4.2%	11.7%
8	4.5	...	3.9
9	22.7	12.1	32.4
10	4.6	2.9	13.1
11	54.5	77.0	38.7
13	...	4.6	...

greatly reduced (for males, 17.07 per cent in right hands, 19.07 per cent in left; for females, 19.76 per cent in right hands and 20.99 per cent in left).

It has long been known that the main line formulae are higher in the right hand than they are in the left; that is, the main lines terminate more distally on the periphery of right hands. The average main line index for 92 mongoloid males was determined and found to be 10.6 for right hands and 10.3 for left hands. The average main line index for 81 mongoloid females was 10.3 for right hands and 9.1 for left. The average main line index for normal European-Americans was found by Cummins to be 9.5 for right hands and 7.8 for left. In both male

and female mongoloids, transversality is somewhat more pronounced in right palms than in left palms, and is also more pronounced than in the palms of normal persons.

It is interesting to note (see Table 2) the termination of main line D at position 13 in 4.6 per cent of mongoloids, whereas Rife (14) found main line D to terminate at position 13 in less than one-fourth of one per cent of normal European-American individuals. This curious piling up on the odd position is another indication of the increased transversality of main lines in mongoloid imbeciles.

TABLE 3

Frequency of commoner main line formulae

FORMULAE	EUROPEAN-AMERICAN (400 palms)	MONGOLOID IMBECILES (346 palms)
11.9.7.-	33.0%	64.1%
11.(o.X.8.)7.-	7.0	9.2
9.7.5.-	16.7	6.9
9.(o.X.8.)-	.7	1.7
7.5.5.-	15.7	.8
All others	21.9	17.2

Table 3 shows the frequency of main line formulation 11.9.7.- in mongoloid imbeciles to be almost twice that of normal persons, further pointing to an increase in the transversality of the main lines in mongoloid imbeciles.

Unfortunately, the number of individuals related to the mongoloids, 24, who were willing to give dermatoglyphic data, is too small to give any significant results. However, there appears to be an increase in patterns in the thenar/first and third interdigital areas of normal individuals related to the mongoloid over those patterns in other normal European-Americans, as well as an increase in the frequency of the termination of main line D at position 11. If any significant similarities exist between the dermatoglyphics of the parent and his mongoloid offspring, they are quite subtle.

Cummins suggests that some of the distinctions in the palmar dermatoglyphics may be associated with the presence of the simian line, a distal transverse flexion crease coursing from the radial to distal margins of the palm. The simian line occurs occasionally in

normal persons, but is quite frequent among mongoloids. My data show 78 cases with the simian line in both hands, 41 cases in one hand only, and 54 cases in which it was altogether absent. The simian line was absent in both hands of 23 individuals related to the mongoloid and was present in one hand only of one of the related individuals.

Although it bears no apparent relation to the dermatoglyphics of mongoloid imbeciles, it is interesting to note that 121 of the 173 mongols showed all the typical features of Mongolism; namely, epicanthus of the eye, fissured tongue, and ear anomalies; 23 showed no epicanthus but had fissured tongue; 16 showed epicanthus only, and 13 showed fissured tongue only. Of the 173 observed, 105 had dark, straight fine textured hair; the remainder showed the common variations in hair form, color, and texture.

Regarding the birth order of the mongoloid in the sibship, 16 were only children, 14 were first in the birth order, and 76 were last. Family size ranged from one child to 13 children, and in no case did I find more than one mongoloid in a sibship, although multiple incidence of Mongolism has been reported by other investigators (Macklin '29, Penrose '32).

The age of the mother at the time of birth of the mongoloid child ranged from 18 to 48, the largest percentage, 7.2, being born to mothers of age 42. I was unable to obtain information regarding the percentage of normal children born to mothers of this same age. However, Workman found no significant correlation between maternal age and pattern expression. She concluded that "whatever may be the cause of the growth disturbance which brings about the distribution of patterns characteristic of the mongoloid hand, it can not be maternal age, as such."

It is desirable to know if the presence and absence of patterns in the five areas of right and left hands of normal persons is independent of the presence and absence of patterns in the pattern areas of each hand of mongoloid imbeciles. Independence may be tested by the method of Bartlett's 2x2x2 tables. The numbers of the 8 cells of the table may be symbolized thus for each pattern area:

	Right hand		Left hand	
	Presence	Absence	Presence	Absence
Normal	*a*	*b*	*a'*	*b'*
Mongol	*c*	*d*	*c'*	*d'*

The value of the deviation, x, common to all 8 cells of the table is determined by solving the equation

$$(a-x)\ (d-x)\ (c'-x)\ (b'-x) = (c+x)\ (b+x)\ (a'+x)\ (d'+x)$$

and chi-square is found by

$$\chi^2 = \left((\mu)-.05\right)^2 \left(\frac{1}{a-x} + \frac{1}{b+x} + \frac{1}{c+x} + \frac{1}{d-x} + \frac{1}{a'+x} + \frac{1}{b'-x} + \frac{1}{c'-x} + \frac{1}{d'+x}\right).$$

Chi-square tests the hypothesis that there is no differential response, or interaction, between the presence and absence of patterns in right and left hands of normal and mongoloid persons. In each case chi-square was non-significant, indicating that the two populations, normal and mongoloid, are independent.

Since the two populations were found to be independent it is convenient to combine the data for each area of each hand into summary fourfold tables and test the hypothesis that there is no difference between the pattern frequencies of normal and mongoloid persons.

The numbers in the cells of the table and in the borders may be symbolized thus:

	Right hand		
	Presence	Absence	Total
Normal	a	b	$a+b$
Mongol	c	d	$c+d$
Total	$a+c$	$b+d$	n

where $n = a+b+c+d$, and

$$\chi^2 = \frac{\left((ad+bc) - n/2\right)^2 n}{(a+b)\ (c+d)\ (a+c)\ (b+d)}$$

The values of chi-square obtained (see Table 4) indicate that the differences in the pattern frequencies between mongoloid imbeciles and normal individuals are highly significant. The statistical analysis not only confirms previous statements regarding the significance of the

departure of the dermatoglyphics of mongoloid imbeciles from those of normal persons, but also gives an estimate of the degree of that departure.

The differences between the palmar configurations of mongoloid imbeciles and normal persons are outstanding and statistically they are highly significant. There is an increase in the pattern frequency in the hypothenar, second, and third interdigital areas, and a decrease in the pattern frequency in the thenar/first, and fourth interdigital areas. Patterns in hypothenar, third, and fourth interdigital areas are more frequent in right hands of normal persons, and in the left

TABLE 4

Significance of differences between mongoloid and normal pattern frequencies by palmar areas

AREA	RIGHT HAND			LEFT HAND		
	χ^2	d.f.	P	χ^2	d.f.	P
Hypothenar	6.45	1	<.01	7.73	1	<.01
Thenar/I	31.31	1	<.01	10.58	1	<.01
II	6.76	1	<.01	13.52	1	<.01
III	60.46	1	<.01	98.07	1	<.01
IV	13.84	1	<.01	5.90	1	<.01

hand patterns are more frequent in the thenar/first and fourth interdigital areas. The occurrence of these patterns in mongoloids might be interpreted as an exaggerated dextral trend.

The aberrancies already demonstrated indicate that the physical distinctions of Mongolism exist as early as the 12th to 14th week of fetal life, the period in which the dermatoglyphics are differentiated. Whatever may be the primary cause of Mongolism it seems to express itself in a retardation of growth affecting many organs.

The question may arise as to whether the dermatoglyphics of mongoloid imbeciles resemble those of normal Mongolian stocks. Cummins ('39) found, by comparing pattern frequencies, that mongoloid imbeciles resemble their own racial stock, European-American, more than they do the Mongolian. Penrose ('32) came to a similar conclusion by comparing the blood group distributions of Mongolian peoples

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and mongoloid imbeciles. It is generally agreed that the trends of the mongoloids represent accentuations of the differences which distinguish Caucasoid peoples from Mongolians (Steggerda '36).

Despite the paucity of the data it is apparent that the dermatoglyphics of the mongol differ from those of his parents and sibs in the same manner as they differ from the dermatoglyphics of other normal European-Americans. On the basis of the small number of prints of individuals related to the mongoloid imbecile it seems clear that their dermatoglyphics would have no obvious value in predicting the chances of a mongoloid birth.

Inasmuch as the dermatoglyphics of mongoloid imbeciles differ significantly from those of normal persons it is suggested that they be used in the diagnostic stigmata of Mongolism. Although further investigation is necessary to show how far the dermatoglyphics could be valuable in diagnosis, they would seem to be of particular value in cases of doubtful diagnosis, for they are as typical of Mongolism as are epicanthus of the eye, fissured tongue, and ear anomalies, the cardinal features used in diagnosis.

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NOTES

A NOTE ON THE SEX RATIO OF ABORTIONS*

IT HAS been known for a long time that among newborn infants boys are slightly more numerous than girls. In a sample of 46 million births registered in Western Europe from 1929 to 1937 the sex ratio was 105.5 males per 100 females for live born infants, 126.7 for stillbirths, and 106.2 for all fetuses reaching viability. Almost identical figures have been found in the United States where, among 14 million births registered from 1941 to 1945, the sex ratio was 105.6 for live births, 124.1 for stillbirths, and 106.0 for the two groups combined. Data for other continents are less reliable but suggest a high degree of uniformity throughout the human race.

Students of human biology have also been interested in the sex ratio at the time of conception, often described as the primary sex ratio. The primary sex ratio is not accessible to direct observation. According to prevailing opinion it is much higher than the sex ratio among viable infants. This opinion is based on statistics of fetal deaths by period of gestation collected in several European and American cities and states. The largest and best known of these series (11, 2) are presented in Table 1. They show a consistent inverse relation between sex ratio and duration of pregnancy. For the four series combined the sex ratio is 114 in the seventh month, 121 in the sixth, 143 in the fifth, 224 in the fourth month, and much higher among the younger embryos. The total number of abortions in the four series is almost 130,000 and from a statistical point of view the significance of the pattern cannot be doubted.

No data are available for the earliest months of pregnancy but several authors have felt justified in extrapolating the pattern of the later

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months and have made very high estimates of the sex ratio among early abortions. From these estimates, and under different assumptions as to the incidence of abortion at each stage of gestation, the primary sex ratio was computed to be 125 by Auerbach (1), at least 130 by Tschuprow (11), 134 by Wedervang (12), 124 by Günther (4), 170 by Crew (3), 146 by Pfaundler (6), and 137 by Lawrence (5) using Ciocco's (2) data. These figures have found wide acceptance and have been the basis of much speculation, especially among German authors, concerning the excess mortality of the male sex.

Unfortunately the determination of sex in a young fetus (8, 10) is not as easy as it would appear to a person who is not familiar with the

TABLE 1

Sex ratio of registered abortions by month of gestation

MONTH OF GESTATION	VIENNA[1] 1893-1910	BUDAPEST[2] 1903-1907	PARIS[1] 1901-1909	UNITED STATES[2] 1925-1934
3rd	727.7[3]			367.3[3]
4th	283.1	231.2	200.5[3]	201.2
5th	153.1	153.9	130.6	139.6
6th	114.9	119.1	122.7	122.7
7th	113.4	117.1	120.7	112.4

[1] From Tschuprow (11). Data for Vienna exclude 1905.
[2] From Ciocco (2). Data from seven states and two cities.
[3] Including a few cases attributed to the preceding month of gestation.

matter. K. M. Wilson (13) has demonstrated conclusively that even trained experts cannot make a reliable diagnosis by inspection of the external genitalia if the embryo has not reached the crown-rump length of 50 mm., corresponding to the end of the eleventh week from the onset of the last menstruation. Even beyond this stage the large clitoris of the female will often be mistaken for a penis by a less experienced observer. As the statistics previously quoted are based upon the statements of medical practitioners without specialized training in embryology, or of midwives, the sex diagnoses for the earlier months cannot be considered reliable.

The only reasonably large series of abortions to which this criticism does not apply is that published by Schultz (7) who studied 1410

embryos and fetuses in the Carnegie Collection. His specimens ranged from the third to the tenth month of gestation with the following sex ratios for the previable period: 108 in the seventh month, 88 in the sixth, 110 in the fifth, 118 in the fourth, and 121 in the third month. The numbers of specimens in each group are not given, but the unweighted mean for the five months is 109 and it appears practically certain the deviations from this mean are statistically not significant.

The present author is indebted to Dr. George W. Corner for his permission to use the records of the Carnegie Collection for a new and more comprehensive investigation into the sex ratio of abortions. The specimens in this collection have been contributed over a number of years by physicians in private practice and in hospitals, most of them from Baltimore. A request for fetuses of specified sex, size, etc., has never been made and it is believed that the collection represents the general run of fetal deaths that come to the attention of the medical profession. The great majority of abortions were unintentional, but in some cases it is known and in others suspected that instrumental manipulation or other interference preceded the expulsion. A few specimens were obtained by operation. The majority of mothers were white women.

From the main catalogue of the Carnegie Collection pertinent information was abstracted in May 1948[1] for 5,667 fetuses of 50-249 mm. crown-rump length, corresponding to the fourth to seventh month of pregnancy (9). The sex of these fetuses had been determined by specially trained technicians under the supervision of the professional staff. This series was supplemented by 120 specimens of 20-49 mm. crown-rump length, approximately corresponding to the third month of pregnancy, which had been sectioned without reference to their external genitalia. The sex of these embryos and younger fetuses was determined by microscopic examination of the gonads. Most of these determinations were made by the late Dr. George L. Streeter. The specimens studied by Schultz are, of course, included in the present series.

Table 2 presents the results of this investigation which fully confirms the earlier findings. Of the 5787 specimens tabulated, 3003 or 51.9 ± 0.7 per cent were males and 2784 or 48.1 per cent females. Expressed in the conventional manner the sex ratio amounted to 107.9 males per 100 females. The ratios for the several months of pregnancy

[1] The most recent entry in the catalogue at this time was 8582.

ranged from 93.5 to 115.6 but none of them was significantly different from the ratio for the whole series. Specifically, no trend toward masculinity was found among the younger specimens.

These findings suggest that the official statistics reproduced in Tabl. 1 are grossly in error. The actual ratio between males and females appears to be about the same for abortions from the third to the seventh

TABLE 2

Sex ratio of specimens in the Carnegie Collection

CROWN-RUMP LENGTH (mm)	MONTH OF GESTATION	NUMBER MALES	NUMBER FEMALES	PERCENTAGE MALES	SEX RATIO
20- 49	3rd	58	62	48.3±4.6	93.5
50- 99	4th	598	555	51.9±1.5	107.7
100-149	5th	771	667	53.6±1.3	115.6
150-199	6th	801	730	52.3±1.3	109.7
200-249	7th	775	770	50.2±1.3	100.6
20-249		3003	2784	51.9±0.7	107.9

month of gestation as it is among viable infants. No excess mortality of males can be demonstrated for this part of pregnancy. The first two months are not accounted for but the data presented here contain no indication that a great difference of mortality between the two sexes exists at that time. There is no evidence for a primary sex ratio materially different from the approximately 106 males per 100 females found among the combined live and stillbirths. Hypotheses based upon a heavy excess mortality of males during the previable period would seem to be in need of revision.

SUMMARY

Among 5787 embryos and fetuses of 20-149 mm. crown-rump length, corresponding to the third to seventh month of pregnancy, the sex ratio was found to be 107.9 males per 100 females. No excess mortality of males during the previable period could be demonstrated.

List of specimens, 20-49 mm. CR length, by sex

Males: 22, 57, 75, 86, 108, 128, 145, 210, 224, 227, 240, 405, 449, 455, 462, 464, 609, 626, 630, 756a, 782, 875, 878, 950, 1008, 1031, 1199, 1318, 1458, 1535, 1591, 1597b, 1686, 1826, 1915, 2206, 2393, 2561, 2779, 3167, 3681, 3735, 4021, 4160, 4193, 4361, 4525, 4570, 4638, 4921, 5154, 5621a, 5762, 6202, 6426, 7254, 7864, 8157.

Females: 6, 10, 45, 95, 199, 211, 362, 368, 382, 417, 460, 632, 635b, 657, 781, 837, 840, 886, 895, 903c, 966, 972, 1153, 1161, 1315, 1358b, 1358f, 1611a, 1894, 1945, 2514, 2937, 3143, 3419, 3527, 3669, 3699, 3881, 4059, 4090, 4148, 4205, 4289, 4304, 4339, 4415, 4475, 4476, 4960, 5017, 5422, 5561, 5596, 5621b, 5725, 6573, 6647, 6701, 6832, 7392, 7425, 8553.

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QUOTATIONS

"PROBABILITY AND LISTERISM"[1]

THE title above was given by the editor of the *Educational Times*[2] to what is perhaps the earliest discussion in English of the problem of comparing small samples from binomial populations. The original problem, proposed by MacAlister[3], ran as follows:

> "Of 10 cases treated by Lister's method, 7 did well and 3 suffered from blood-poisoning; of 14 cases treated with ordinary dressings, 9 did well and 5 had blood-poisoning; what are the odds that the success of Lister's method was due to chance?"

The wording of the quotation would not now be altogether acceptable to statisticians; but we need not quarrel with the phraseology. MacAlister is clearly asking whether, with no real difference between the methods, chance could have given the observed result. It is interesting to see what responses the question brought.

The first solution was offered by Dr. Macfarlane, F.R.S.E.[4], and may be quoted in full:

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[1] From the Department of Biostatistics, (Paper No. 244), School of Hygiene and Public Health, The Johns Hopkins University, Baltimore, Md.

[2] *Educational Times Reprints*, 37: 40-42; 59-61; 77-80; 92. 1882.

[3] MacAlister, Donald (1854-1934). Senior wrangler and first Smith's prizeman, Cambridge 1877. M.B. 1881. M.D. 1884. F.R.C.P. 1886. 1881-1907, Director Medical Studies, Glasgow; 1907-1929, Principal and Vice Chancellor, Glasgow; 1929-1934, Chancellor, Glasgow.

Best known to statisticians for his note on "The law of the geometric mean" (*Proc. R. Soc.*, 29: 367. 1879), written at Galton's suggestion.

[4] "Alexander Macfarlane (1851-1913) of the University of Edinburgh, later of the University of Texas and of Lehigh University, served as secretary of the International Association for Promoting the Study of Quaternions and Allied Systems of Mathematics, and was its president at the time of his death." (Cajori, History of Mathematics, 2d ed., 1919.)

"Let p denote the chance of a case treated by Lister's method doing well, and q the chance of a case treated with ordinary dressings doing well, then $p = 7/10$ and $q = 9/14$. But Lister's method consists in the ordinary dressings with the additional use of an antiseptic; hence the effect of the antiseptic is $p—q$, that is, $2/35$. Hence the odds that in a given case the success of Lister's method is not due to the characteristic part of it is

$$\frac{q}{p—q}, \text{ that is, } \frac{45}{4}."$$

This solution is likely to puzzle a modern reader as much as it did Dr. MacAlister. The fact is that Macfarlane has misunderstood the question. He has assumed that we were given the true probabilities, and has calculated the odds that a patient recovering after treatment by Lister's method would equally have recovered under the ordinary treatment. The solution is correct for this interpretation of the problem. The misinterpretation is interesting, and apparently indicates that the notion of calculating probabilities for small samples was relatively unfamiliar.

MacAlister does not give the derivation of his own solution, which actually he owes, as he says, to Liebermeister[5] (about whom something will be said below). He gives the following:

Suppose:

of $a + b$ balls drawn from (with replacement) A, a are white;
of $p + q$ balls drawn from B, p are white.

Further suppose

$$\frac{a}{a+b} > \frac{p}{p+q}$$

Let P = the probability that the proportion of white balls in B is less than that in A.

Then:

$$P = \frac{(a+b+1)!\,(p+q+1)!\,(a+p+1)!\,(b+q+1)!}{a!\,(b+1)!\,(p+1)!\,q!\,(a+b+p+q+2)!} \times S,$$

$$S = 1 + \frac{a}{b+2} \cdot \frac{q}{p+2} + \frac{a(a—1)}{(b+2)(b+3)} \frac{q(q—1)}{(p+2)(p+3)} + \cdots,$$

$$1 - P = \frac{(a+b+1)!\,(p+q+1)!\,(a+p+1)!\,(b+q+1)!}{(a+1)!\,b!\,p!\,(q+1)!\,(a+b+p+q+2)!} \times S',$$

[5] Liebermeister, Carl (1833-1901). Professor of pathology and director of the medical clinic, Basel, 1865-1871. Same post, Tübingen, 1871-1901.

$$S = 1 + \frac{b}{a+2} \cdot \frac{p}{q+2} + \frac{b(b-1)}{(a+2)(a+3)} \frac{p(p-1)}{(p+2)(p+3)} + \ldots\ldots$$

In the particular numerical case, $a = 5$, $b = 9$, $p = 3$, $q = 7$, and

$$1 - P = .40175, \qquad P = .59825,$$

so that, says MacAlister, "we may wager nearly 3 to 2 that the difference in the results is not due to chance."

We shall discuss the merits of the solution later. Meanwhile some of MacAlister's remarks are worth quoting:

> "This is, in fact, our only means of testing the relative values of different methods; by their fruits we know them. Our problem is—from our small number of observed cases to gather what we can about the outcome of the two methods if they were applied on the largest possible scale. It is to learn what we can from a basket of apples concerning the whole contents of the orchard.
>
> "The authorities on Probability give formulae which only apply to 'samples' of great magnitude; a mere basketful is nothing to them. But it may fairly be urged that even a basketful gives *some* information. A buyer would not be held unwise who, knowing nothing but what the samples told him, should prefer to invest in the orchard whence came the better basket. He might reasonably ask the mathematician to appraise the amount of information the samples gave him, even though the samples were not very large."

The solution offered by MacAlister was criticized by Hugh MacColl[6], on the ground that the unwarranted assumption has been made that, before the experiment, "it is an even chance whether Lister's method has any advantage." This is, of course, true; the solution, as we shall see later, is a Bayes' theorem solution, and subject to the criticisms that can be made of any Bayesian argument.

MacAlister, in reply to this criticism, says that the *a priori* chance is "a very important factor, when you know it. When you don't know it, does Mr. MacColl mean to say that the problem cannot have any solution?" To this MacColl suggests that we "denote the *a priori* chance by some symbol a, and wait patiently for further knowledge before we committed ourselves to any numerical result?"

---

[6] "Departing from the tradition of Boole, MacColl (1837-1909, England), all but hit upon the concept of a propositional function. . . . it seems just to say that although MacColl may not have seen clearly where he was going, he had set foot on a promising road to what, in his day, was the future of mathematical logic." (E. T. Bell, Development of Mathematics, pp. 520-521, *New York*, 1940.)

One may perhaps question the relevance of the suggestion to "wait patiently for further information" about the *a priori* probability. Surely it is of the essence of the problem that the solution is to be found from the information at hand, and not from "further information." Indeed, it is not easy to see how further information could affect the *a priori* probability in any case.

A solution was submitted by Whitworth[7] which is of some interest, though I think it is clearly wrong. He says:

> "Let p be the probability of success under the old treatment and μp the probability of success under the new (or Lister's) treatment. Then the *a priori* probability of the observed event was
>
> $$P = p^{8}(1-p)^{4}(\mu p)^{7}(1-\mu p)^{3},$$
>
> and μ may have any values between 0 and p^{-1}. If we are asked what is the chance that $\mu = 1$, the answer is obviously zero, for the number of possible values that μ could have between the above-named limits is indefinitely large, and the chance of any particular one is indefinitely small. But if we are asked what is the chance that $\mu < 1$ (that is, that Lister's method gives no advantage, but may be disadvantageous), the probability is
>
> $$\frac{\sum_{\mu=0}^{\mu=1} P}{\sum_{\mu=0}^{\mu=1/p} P} = \frac{\int_{0}^{1} \mu^{7}(1-\mu p)^{3}d\mu}{\int_{0}^{1/p} \mu^{7}(1-\mu p)^{3}d\mu}$$
>
> $$= 165p^{8} - 440p^{9} + 396p^{10} - 120p^{11};$$
>
> and if our knowledge is limited to the statement of the question, we must put $p = 9/14$. This makes the chance $= .0078$ approx. The complementary chance is therefore .9922, and the odds are about 229 to 2 in favor of Lister's method being advantageous."

These are surprising odds. Actually, they arise from a numerical slip. "The chance" should be .418 rather than .0078. (Whitworth seems to have calculated $(9/14)^{11}$, which is .00775.) Of rather more interest, I think, is the fact that for no very clear reason Whitworth

[7] Whitworth, Rev. W. A. (1840-1905). St. Johns College, Cambridge, 1862; ordained 1865; author Modern Analytical Geometry, 1866; Choice and Chance, 1867, with later editions; DCC Exercises in Choice and Chance, 1897; the Real Presence, 1893; Worship in the Christian Church, 1901. (Choice and Chance is a classic; it was reprinted by G. E. Stechert & Co., *New York*, 1927.)

applies Bayes' theorem to one quantity, μ, but not to both μ and p. It would seem more logical to set the probability sought as

$$\frac{\int_0^1 \int_0^1 p^9(1-p)^5(\mu p)^7(1-\mu p)^3 \, d\mu \, dp}{\int_0^1 \int_0^{1/p} p^9(1-p)^5(\mu p)^7(1-\mu p)^3 \, d\mu \, dp},$$

in which we have assumed that *a priori* both μ and p are equally likely to lie anywhere in their respective possible ranges. To integrate, we may substitute

$$x = \mu p, \qquad dx = p \, d\mu,$$

getting

$$\frac{\int_0^1 \int_0^{1/p} p^8(1-p)^5 x^7 (1-x)^3 \, dx \, dp}{\int_0^1 \int_0^1 p^8(1-p)^5 x^7 (1-x)^3 \, dx \, dp}.$$

It will now be seen that this is of the same form as Liebermeister's integral (2) below, except that the exponent of p is reduced by unity.

The point of interest here is that we have another illustration of the difficulties which arise in the application of Bayes' theorem. Liebermeister and MacAlister start with unknown probabilities p and x; Whitworth starts with unknowns p and μ. There is no obvious reason why one choice is not as valid as the other, yet they lead to different results. There is nothing here to surprise anyone familiar with the literature on Bayes' theorem.

It seems likely that Whitworth was not altogether satisfied with his solution, for he goes on:

> "The above measures the chance that Lister's method is some good, as against the position that it may be useless or disadvantageous.
>
> "But P is a maximum when $\mu = 7/10p$. This is therefore the *most likely* value of μ; in other words, 7/10 is a more probable value than any other that might be alleged for the chance of success under Lister's method. Now, if Lister were to maintain that with his method the chance is 7/10, while his opponents allege that Lister's treatment makes no difference whatever, and that the chance of success is 2/3 (not 9/14, for if they claim that Lister's treatment makes no difference they must count *all* the observed cases in estimating the chance of success), then the odds in favour of

Lister's claim as against the claim of his opponents are expressed by the ratio of the two values of P, (1) when $p = 9/14$, $\mu p = 7/10$, and (2) when $p = \mu p = 2/3$. This ratio is $3^{46} : 2^{23} \cdot 7^{7} \cdot 10^{8}$. That is to say, the odds are as 29543 : 28296 in favour of Lister having the precise advantage denoted by $\mu p = 7/10$ as against the statement that his treatment makes no difference. To sum up my results:

"(α) The chance (from the observed cases only) that Lister's method should precisely make no difference, is less than any assignable chance. The question as stated can only mean this, and the only true answer is zero.

"(β) The odds that Lister's method is beneficial, as against the position that it is either useless or injurious, are about 114 : 1.

"(γ) The odds in favour of the statement that Lister's treatment will succeed in 7 cases out of 10, as against the position that his treatment makes no difference, are about 21 : 20."

One final point ought to be made. It has doubtless not escaped the careful reader. If the problem is set up as a 2 x 2 table, we have:

	Recovered	Blood poisoning	Total
Lister	7	3	10
Ordinary	9	5	14
Total	16	8	24

But now it will be seen that with these marginal totals, it is not possible to get a closer approach to equality between the treatments.

As was remarked above, the solution offered by MacAlister is taken from Liebermeister. It is therefore of some interest to see what Liebermeister himself had to say[8].

The mathematical problem which Liebermeister solves is the following:

A sample from population 1 has given a failures and b successes; a sample from population 2 has given p failures and q successes. We have

$$\frac{p}{p+q} < \frac{a}{a+b} \quad .$$

Required, the probability that the true proportion of failures in population 2 is less than that in population 1.

[8] Liebermeister, C. Ueber Wahrscheinlichkeitsrechnung in Anwendung auf Therapeutische Statistik. Sammlung Klinischer Vorträge. ed. R. Volkmann, No. 110 (Innere Medicin No. 39), pp. 935-962. 1877.

As we might expect, the solution is based on Bayes' theorem. Letting α and β be the probabilities of failure in populations 1 and 2, we have as the probability of the observed result, given α and β,

$$\frac{(a+b)!}{a!\,b!}\,\frac{(p+q)!}{p!\,q!}\,\alpha^{a}(1-\alpha)^{b}\,\beta^{p}(1-\beta)^{q}. \qquad (1)$$

If now we make the Bayesian assumption that α and β are independently equally likely to have any values between 0 and 1, we have to integrate (1) over the set of values for which $\alpha > \beta$, and divide by the integral of (1) over the unit square:

$$P = \frac{\int_0^1 \int_0^{\alpha} \alpha^{a}(1-\alpha)^{b}\,\beta^{p}(1-\beta)^{q}\,d\beta\,d\alpha}{\int_0^1 \int_0^1 \alpha^{a}(1-\alpha)^{b}\,\beta^{p}(1-\beta)^{q}\,d\beta\,d\alpha}. \qquad (2)$$

The result is expressed by Liebermeister in various forms, of which those quoted above from MacAlister are two. Of particular interest is the second,

$$1 - P = \frac{(a+b+1)!}{(a+1)!\,b!}\,\frac{(p+q+1)!}{p!\,(q+1)!}\,\frac{(a+p+1)!\,(b+q+1)!}{(a+b+p+q+2)!} \times$$

$$\left\{1 + \frac{b}{a+2}\,\frac{p}{q+2} + \frac{b(b-1)}{(a+2)\,(a+3)}\,\frac{p(p-1)}{(q+2)\,(q+3)} + \ldots\right\} \qquad (3)$$

which invites comparison with Fisher's exact treatment of the 2 x 2 table (§ 21.02). Fisher's expression is

$$\frac{(a+b)!\,(p+q)!\,(a+p)!\,(b+q)!}{a!\,b!\,p!\,q!\,(a+b+p+q)!}\left\{1 + \frac{b}{a+1}\,\frac{p}{q+1} + \right.$$

$$\left. \frac{b(b-1)}{(a+1)\,(a+2)}\,\frac{p(p-1)}{(q+1)\,(q+2)} \ldots\right\}. \qquad (4)$$

It will be observed that (3) and (4) differ only in that a and q in (4) are replaced by $a+1$ and $q+1$ in (3). That is, Liebermeister's probability is the same that Fisher would calculate from a table with the frequencies on one diagonal increased by unity. It is further evident that this will mean, especially in small samples, that Liebermeister's

result will give smaller values of $1-P$ (and hence apparently stronger indications of significance).

I think it is clear that few of us today would use the Liebermeister solution. The interest lies in his recognition of the problem.

Some of Liebermeister's remarks may be of interest, as showing that he had a clear understanding of what statistical methods could and could not do for the practitioner:

> (p. 936) . . . "Naturally, our problem is by no means solved with the settling of this mathematical or formal part. On the contrary, there then arises the question whether the two sets of observations are really comparable in every respect except the difference of treatment, whether perhaps there may have been a decisive change in the character of the disease or the cause of the disease, whether a change in one of the many other circumstances on which the result depends may not have determined the difference in the observed results. It would evidently be a mistake to seek the answer to this question in the mathematical analysis. The theory of probability gives us, accurately, with what probability we may assert that there has been a difference in the conditions determining the result; but this theory is wholly incapable of saying anything as to the origin of this difference. The investigation of causes, and specifically the question whether the cause is to be sought in the difference in treatment or in differences of some other circumstance is the affair of clinical analysis."
>
> (p. 952) . . . "6) Finally, we may calculate an example which will also show the extent to which the meaning of the usually applied formulae has occasionally been misunderstood by medical statisticians. In the exposition of the mathematical foundations of medical statistics by Hirshberg the following example is given to illustrate the theorem that no weight is to be given to small variations in statistical ratios: 'If in a series of 300 cases of a disease, for example typhoid, we observe a mortality of 22%, and in a second series of 300 cases of typhoid a mortality of 16%, the true mortality in both series may be the same; indeed, this is highly probable.' Such a statement does not in fact represent the usual view of the practical doctor, who would certainly consider the results of the second series essentially better; it also does not represent the result of an unprejudiced examination. Such would certainly allow that the difference in the ratios is not sufficient to draw sure conclusions from not very large numbers; the *possibility* that the difference is a chance difference would not be doubted; but that this is *probable* would certainly not be obvious to common sense. Now the Poisson formula, rightly applied, gives a probability of .9397 for the assumption that the lower mortality in the second series is not due to chance, but arises from some constant cause; that is, one can wager more than 15 to 1 that the lower mortality is not accidental. Of course, the question *what* has caused the lower mortality—whether a change in treatment, a change in the character of the epidemic, or some other change in conditions—is not one for mathematics but for clinical

analysis. And also odds of 15 to 1 are far from absolute certainty; they are not as long as one might wish, if one is to draw important conclusions, and in particular not as long as Gavarret's formula requires; but they are certainly not meaningless."

CHARLES P. WINSOR